### COMPARISON OF PROPERTIES OF DOT PRODUCT AND CROSS PRODUCT

<table>
<thead>
<tr>
<th>DOT PRODUCT</th>
<th>CROSS PRODUCT</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mathbf{u} \cdot \mathbf{v} = u_1v_1 + u_2v_2 + u_3v_3 )</td>
<td>( \mathbf{u} \times \mathbf{v} = \left\langle (u_2v_3 - u_3v_2), (u_3v_1 - u_1v_3), (u_1v_2 - u_2v_1) \right\rangle )</td>
</tr>
<tr>
<td>( \mathbf{u} \cdot \mathbf{v} = |\mathbf{u}| |\mathbf{v}| \cos \theta )</td>
<td>( |\mathbf{u} \times \mathbf{v}| = |\mathbf{u}| |\mathbf{v}| \sin \theta )</td>
</tr>
<tr>
<td>( \mathbf{u} \times \mathbf{v} ) does not have a &quot;direction&quot; because it is a scalar, not a vector</td>
<td>Direction of ( \mathbf{u} \times \mathbf{v} ) is determined by right hand rule: Curl fingers of right hand from ( \mathbf{u} ) to ( \mathbf{v} ); thumb points in direction of ( \mathbf{u} \times \mathbf{v} )</td>
</tr>
</tbody>
</table>

### PROPERTIES OF DOT PRODUCT

- Commutative: \( \mathbf{u} \cdot \mathbf{v} = \mathbf{v} \cdot \mathbf{u} \)
- Distributive over addition: \( \mathbf{u} \cdot (\mathbf{v} + \mathbf{w}) = \mathbf{u} \cdot \mathbf{v} + \mathbf{u} \cdot \mathbf{w} \)
- \( \mathbf{c} \cdot (\mathbf{u} \cdot \mathbf{v}) = (\mathbf{c} \cdot \mathbf{u}) \cdot \mathbf{v} = \mathbf{u} \cdot (\mathbf{c} \cdot \mathbf{v}) \)

### PROPERTIES OF CROSS PRODUCT

- Anticommutative: \( \mathbf{u} \times \mathbf{v} = - \mathbf{v} \times \mathbf{u} \)
- Distributive over addition: \( \mathbf{u} \times (\mathbf{v} + \mathbf{w}) = \mathbf{u} \times \mathbf{v} + \mathbf{u} \times \mathbf{w} \)
- \( \mathbf{c} \times (\mathbf{u} \times \mathbf{v}) = (\mathbf{c} \times \mathbf{u}) \times \mathbf{v} = \mathbf{u} \times (\mathbf{c} \times \mathbf{v}) \)
- \( \mathbf{i} \times \mathbf{j} = \mathbf{k} \quad \mathbf{j} \times \mathbf{i} = \mathbf{k} \quad \mathbf{k} \times \mathbf{i} = \mathbf{j} \)

### APPLICATIONS OF DOT PRODUCT

- \( \cos \theta = \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\| \|\mathbf{v}\|} \) allows us to find the angle between \( \mathbf{u} \) and \( \mathbf{v} \)

\[
\text{proj}_\mathbf{v} \mathbf{u} = \left( \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{v}\|^2} \right) \mathbf{v} = \left( \frac{\mathbf{u} \cdot \mathbf{v}}{\mathbf{v} \cdot \mathbf{v}} \right) \mathbf{v}
\]

\[
\text{orth}_\mathbf{u} \mathbf{v} = \mathbf{u} - \text{proj}_\mathbf{v} \mathbf{u}
\]

- Work = \( \mathbf{F} \cdot \mathbf{D} \)
  - U.S.: units of work are foot-pounds
  - Metric: units of work are joules

### APPLICATIONS OF CROSS PRODUCT

- Use \( \mathbf{u} \times \mathbf{v} \) when you need to find a vector that is orthogonal to both \( \mathbf{u} \) and \( \mathbf{v} \)

\[
\text{area of parallelogram defined by vectors } \mathbf{u} \text{ and } \mathbf{v} = \|\mathbf{u} \times \mathbf{v}\|
\]

\[
\text{area of triangle defined by vectors } \mathbf{u} \text{ and } \mathbf{v} = \frac{1}{2}\|\mathbf{u} \times \mathbf{v}\|
\]

- Cross product is useful when finding the equation of a plane that contains two given vectors.
  - (This will be covered in section 11.4)