1. The magnitude of the force of one particle on the other is given by $F = Gm_1m_2/r^2$, where m_1 and m_2 are the masses, r is their separation, and G is the universal gravitational constant. We solve for r:

$$r = \sqrt{\frac{Gm_1m_2}{F}} = \sqrt{\frac{\left(6.67\times10^{-11}\,\mathrm{N\cdot m^2/kg^2}\right)\left(5.2\,\mathrm{kg}\right)\left(2.4\,\mathrm{kg}\right)}{2.3\times10^{-12}\,\mathrm{N}}} = 19\,\mathrm{m}.$$

2. We use subscripts s, e, and m for the Sun, Earth and Moon, respectively. Plugging in the numerical values (say, from Appendix C) we find

$$\frac{F_{sm}}{F_{em}} = \frac{Gm_s m_m / r_{sm}^2}{Gm_e m_m / r_{em}^2} = \frac{m_s}{m_e}\left(\frac{r_{em}}{r_{sm}}\right)^2 = \frac{1.99\times10^{30}\ \text{kg}}{5.98\times10^{24}\ \text{kg}}\left(\frac{3.82\times10^{8}\ \text{m}}{1.50\times10^{11}\ \text{m}}\right)^2 = 2.16.$$

3. The gravitational force between the two parts is

$$F = \frac{Gm(M-m)}{r^2} = \frac{G}{r^2}\left(mM - m^2\right)$$

which we differentiate with respect to *m* and set equal to zero:

$$\frac{dF}{dm} = 0 = \frac{G}{r^2}(M - 2m) \Rightarrow M = 2m \,.$$

This leads to the result $m/M = 1/2$.

4. The gravitational force between you and the moon at its initial position (directly opposite of Earth from you) is

$$F_0 = \frac{GM_m m}{(R_{ME} + R_E)^2}$$

where M_m is the mass of the moon, R_{ME} is the distance between the moon and the Earth, and R_E is the radius of the Earth. At its final position (directly above you), the gravitational force between you and the moon is

$$F_1 = \frac{GM_m m}{(R_{ME} - R_E)^2}.$$

(a) The ratio of the moon's gravitational pulls at the two different positions is

$$\frac{F_1}{F_0} = \frac{GM_m m/(R_{ME} - R_E)^2}{GM_m m/(R_{ME} + R_E)^2} = \left(\frac{R_{ME} + R_E}{R_{ME} - R_E}\right)^2 = \left(\frac{3.82\times10^8\ \text{m} + 6.37\times10^6\ \text{m}}{3.82\times10^8\ \text{m} - 6.37\times10^6\ \text{m}}\right)^2 = 1.06898.$$

Therefore, the increase is 0.06898, or approximately, 6.9%.

(b) The change of the gravitational pull may be approximated as

$$F_1 - F_0 = \frac{GM_m m}{(R_{ME} - R_E)^2} - \frac{GM_m m}{(R_{ME} + R_E)^2} \approx \frac{GM_m m}{R_{ME}^2}\left(1 + 2\frac{R_E}{R_{ME}}\right) - \frac{GM_m m}{R_{ME}^2}\left(1 - 2\frac{R_E}{R_{ME}}\right) = \frac{4GM_m m R_E}{R_{ME}^3}.$$

On the other hand, your weight, as measured on a scale on Earth is

$$F_g = mg_E = \frac{GM_E m}{R_E^2}.$$

Since the moon pulls you "up," the percentage decrease of weight is

$$\frac{F_1 - F_0}{F_g} = 4\left(\frac{M_m}{M_E}\right)\left(\frac{R_E}{R_{ME}}\right)^3 = 4\left(\frac{7.36\times10^{22}\ \text{kg}}{5.98\times10^{24}\ \text{kg}}\right)\left(\frac{6.37\times10^6\ \text{m}}{3.82\times10^8\ \text{m}}\right)^3 = 2.27\times10^{-7} \approx (2.3\times10^{-5})\%.$$

5. We require the magnitude of force (given by Eq. 13-1) exerted by particle C on A be equal to that exerted by B on A. Thus,

$$\frac{Gm_A m_C}{r^2} = \frac{Gm_A m_B}{d^2} \ .$$

We substitute in $m_B = 3m_A$ and $m_B = 3m_A$, and (after canceling "m_A") solve for r. We find $r = 5d$. Thus, particle C is placed on the x axis, to left of particle A (so it is at a negative value of x), at $x = –5.00d$.

6. Using $F = GmM/r^2$, we find that the topmost mass pulls upward on the one at the origin with 1.9×10^{-8} N, and the rightmost mass pulls rightward on the one at the origin with 1.0×10^{-8} N. Thus, the (x, y) components of the net force, which can be converted to polar components (here we use magnitude-angle notation), are

$$\vec{F}_{\text{net}} = \left(1.04\times10^{-8}, 1.85\times10^{-8}\right) \Rightarrow \left(2.13\times10^{-8} \angle\, 60.6^\circ\right).$$

(a) The magnitude of the force is 2.13×10^{-8} N.

(b) The direction of the force relative to the $+x$ axis is 60.6°.

7. At the point where the forces balance $GM_e m / r_1^2 = GM_s m / r_2^2$, where M_e is the mass of Earth, M_s is the mass of the Sun, m is the mass of the space probe, r_1 is the distance from the center of Earth to the probe, and r_2 is the distance from the center of the Sun to the probe. We substitute $r_2 = d - r_1$, where d is the distance from the center of Earth to the center of the Sun, to find

$$\frac{M_e}{r_1^2} = \frac{M_s}{(d - r_1)^2}.$$

Taking the positive square root of both sides, we solve for r_1. A little algebra yields

$$r_1 = \frac{d\sqrt{M_e}}{\sqrt{M_s} + \sqrt{M_e}} = \frac{(150\times10^9\ \text{m})\sqrt{5.98\times10^{24}\ \text{kg}}}{\sqrt{1.99\times10^{30}\ \text{kg}} + \sqrt{5.98\times10^{24}\ \text{kg}}} = 2.60\times10^8\ \text{m}.$$

Values for M_e, M_s, and d can be found in Appendix C.

8. The gravitational forces on m_5 from the two 5.00g masses m_1 and m_4 cancel each other. Contributions to the net force on m_5 come from the remaining two masses:

$$F_{\text{net}} = \frac{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(2.50\times10^{-3}\ \text{kg}\right)\left(3.00\times10^{-3}\ \text{kg} - 1.00\times10^{-3}\ \text{kg}\right)}{\left(\sqrt{2}\times10^{-1}\ \text{m}\right)^2}$$

$$= 1.67\times10^{-14}\ \text{N}.$$

The force is directed along the diagonal between m_2 and m_3, towards m_2. In unit-vector notation, we have

$$\vec{F}_{\text{net}} = F_{\text{net}}(\cos 45°\hat{\text{i}} + \sin 45°\hat{\text{j}}) = (1.18\times10^{-14}\,\text{N})\,\hat{\text{i}} + (1.18\times10^{-14}\,\text{N})\,\hat{\text{j}}$$

9. The gravitational force from Earth on you (with mass *m*) is

$$F_g = \frac{GM_E m}{R_E^2} = mg$$

where $g = GM_E / R_E^2 = 9.8\ \text{m/s}^2$. If *r* is the distance between you and a tiny black hole of mass $M_b = 1\times 10^{11}\ \text{kg}$ that has the same gravitational pull on you as the Earth, then

$$F_g = \frac{GM_b m}{r^2} = mg.$$

Combining the two equations, we obtain

$$mg = \frac{GM_E m}{R_E^2} = \frac{GM_b m}{r^2} \Rightarrow r = \sqrt{\frac{GM_b}{g}} = \sqrt{\frac{(6.67\times 10^{-11}\ \text{m}^3/\text{kg}\cdot\text{s}^2)(1\times 10^{11}\ \text{kg})}{9.8\ \text{m/s}^2}} \approx 0.8\ \text{m}.$$

10. (a) We are told the value of the force when particle C is removed (that is, as its position x goes to infinity), which is a situation in which any force caused by C vanishes (because Eq. 13-1 has r^2 in the denominator). Thus, this situation only involves the force exerted by A on B:

$$\frac{Gm_A\, m_B}{(0.20\text{ m})^2} = 4.17 \times 10^{-10}\text{ N}\,.$$

Since $m_B = 1.0$ kg, then this yields $m_A = 0.25$ kg.

(b) We note (from the graph) that the net force on B is zero when $x = 0.40$ m. Thus, at that point, the force exerted by C must have the same magnitude (but opposite direction) as the force exerted by A (which is the one discussed in part (a)). Therefore

$$\frac{Gm_C\, m_B}{(0.40\text{ m})^2} = 4.17 \times 10^{-10}\text{ N} \quad \Rightarrow \quad m_C = 1.00\text{ kg}.$$

11. (a) The distance between any of the spheres at the corners and the sphere at the center is

$$r = \ell / 2\cos 30° = \ell / \sqrt{3}$$

where ℓ is the length of one side of the equilateral triangle. The net (downward) contribution caused by the two bottom-most spheres (each of mass m) to the total force on m_4 has magnitude

$$2F_y = 2\left(\frac{Gm_4 m}{r^2}\right)\sin 30° = 3\frac{Gm_4 m}{\ell^2}.$$

This must equal the magnitude of the pull from M, so

$$3\frac{Gm_4 m}{\ell^2} = \frac{Gm_4 m}{\left(\ell/\sqrt{3}\right)^2}$$

which readily yields $m = M$.

(b) Since m_4 cancels in that last step, then the amount of mass in the center sphere is not relevant to the problem. The net force is still zero.

12. All the forces are being evaluated at the origin (since particle A is there), and all forces (except the net force) are along the location-vectors $\vec{r}$ which point to particles B and C. We note that the angle for the location-vector pointing to particle B is 180º – 30.0º = 150º (measured ccw from the $+x$ axis). The component along, say, the x axis of one of the force-vectors $\vec{F}$ is simply Fx/r in this situation (where F is the magnitude of $\vec{F}$). Since the force itself (see Eq. 13-1) is inversely proportional to r^2 then the aforementioned x component would have the form $GmMx/r^3$; similarly for the other components. With m_A = 0.0060 kg, m_B = 0.0120 kg, and m_C = 0.0080 kg, we therefore have

$$F_{\text{net}\,x} = \frac{Gm_A m_B x_B}{r_B^3} + \frac{Gm_A m_C x_C}{r_C^3} = (2.77 \times 10^{-14}\,\text{N})\cos(-163.8^\circ)$$

and

$$F_{\text{net}\,y} = \frac{Gm_A m_B y_B}{r_B^3} + \frac{Gm_A m_C y_C}{r_C^3} = (2.77 \times 10^{-14}\,\text{N})\sin(-163.8^\circ)$$

where $r_B = d_{AB}$ = 0.50 m, and $(x_B, y_B) = (r_B\cos(150^\circ), r_B\sin(150^\circ))$ (with SI units understood). A fairly quick way to solve for r_C is to consider the vector difference between the net force and the force exerted by A, and then employ the Pythagorean theorem. This yields r_C = 0.40 m.

(a) By solving the above equations, the x coordinate of particle C is $x_C = -0.20$ m.

(b) Similarly, the y coordinate of particle C is $y_C = -0.35$ m.

13. If the lead sphere were not hollowed the magnitude of the force it exerts on m would be $F_1 = GMm/d^2$. Part of this force is due to material that is removed. We calculate the force exerted on m by a sphere that just fills the cavity, at the position of the cavity, and subtract it from the force of the solid sphere.

The cavity has a radius $r = R/2$. The material that fills it has the same density (mass to volume ratio) as the solid sphere. That is $M_c/r^3 = M/R^3$, where M_c is the mass that fills the cavity. The common factor $4\pi/3$ has been canceled. Thus,

$$M_c = \left(\frac{r^3}{R^3}\right)M = \left(\frac{R^3}{8R^3}\right)M = \frac{M}{8}.$$

The center of the cavity is $d - r = d - R/2$ from m, so the force it exerts on m is

$$F_2 = \frac{G(M/8)m}{(d - R/2)^2}.$$

The force of the hollowed sphere on m is

$$\begin{aligned} F = F_1 - F_2 &= GMm\left(\frac{1}{d^2} - \frac{1}{8(d-R/2)^2}\right) = \frac{GMm}{d^2}\left(1 - \frac{1}{8(1-R/2d)^2}\right) \\ &= \frac{(6.67\times10^{-11}\ \text{m}^3/\text{s}^2\cdot\text{kg})(2.95\ \text{kg})(0.431\ \text{kg})}{(9.00\times10^{-2}\,\text{m})^2}\left(1 - \frac{1}{8[1-(4\times10^{-2}\,\text{m})/(2\cdot9\times10^{-2}\,\text{m})]^2}\right) \\ &= 8.31\times10^{-9}\,\text{N}. \end{aligned}$$

14. Using Eq. 13-1, we find

$$\vec{F}_{AB} = \frac{2Gm_A^2}{d^2}\,\hat{j} \qquad \text{and} \qquad \vec{F}_{AC} = -\frac{4Gm_A^2}{3d^2}\,\hat{i} \; .$$

Since the vector sum of all three forces must be zero, we find the third force (using magnitude-angle notation) is

$$\vec{F}_{AD} = \frac{Gm_A^2}{d^2}\,(2.404 \;\angle\; -56.3^\circ) \; .$$

This tells us immediately the direction of the vector $\vec{r}$ (pointing from the origin to particle D), but to find its magnitude we must solve (with $m_D = 4m_A$) the following equation:

$$2.404\left(\frac{Gm_A^2}{d^2}\right) = \frac{Gm_A m_D}{r^2} \; .$$

This yields $r = 1.29d$. In magnitude-angle notation, then, $\vec{r} = (1.29 \;\angle\; -56.3^\circ)$, with SI units understood. The "exact" answer without regard to significant figure considerations is

$$\vec{r} = \left(2\sqrt{\frac{6}{13\sqrt{13}}} \, , \, -3\sqrt{\frac{6}{13\sqrt{13}}} \right) .$$

(a) In (x, y) notation, the x coordinate is $x = 0.716d$.

(b) Similarly, the y coordinate is $y = -1.07d$.

15. All the forces are being evaluated at the origin (since particle A is there), and all forces are along the location-vectors $\vec{r}$ which point to particles B, C and D. In three dimensions, the Pythagorean theorem becomes $r = \sqrt{x^2 + y^2 + z^2}$. The component along, say, the x axis of one of the force-vectors $\vec{F}$ is simply Fx/r in this situation (where F is the magnitude of $\vec{F}$). Since the force itself (see Eq. 13-1) is inversely proportional to r^2 then the aforementioned x component would have the form $GmMx/r^3$; similarly for the other components. For example, the z component of the force exerted on particle A by particle B is

$$\frac{Gm_A m_B z_B}{r_B^{\,3}} = \frac{Gm_A(2m_A)(2d)}{((2d)^2 + d^2 + (2d)^2)^3} = \frac{4Gm_A^{\,2}}{27\,d^2}\,.$$

In this way, each component can be written as some multiple of $Gm_A^{\,2}/d^2$. For the z component of the force exerted on particle A by particle C, that multiple is $-9\sqrt{14}\,/196$. For the x components of the forces exerted on particle A by particles B and C, those multiples are $4/27$ and $-3\sqrt{14}\,/196$, respectively. And for the y components of the forces exerted on particle A by particles B and C, those multiples are $2/27$ and $3\sqrt{14}\,/98$, respectively. To find the distance r to particle D one method is to solve (using the fact that the vector add to zero)

$$\left(\frac{Gm_A m_D}{r^2}\right)^2 = [(4/27 - 3\sqrt{14}\,/196)^2 + (2/27 + 3\sqrt{14}\,/98)^2 + (4/27 - 9\sqrt{14}\,/196)^2]\left(\frac{Gm_A^{\,2}}{d^2}\right)^2$$

(where $m_D = 4m_A$) for r. This gives $r = 4.357d$. The individual values of x, y and z (locating the particle D) can then be found by considering each component of the $Gm_A m_D/r^2$ force separately.

(a) The x component of $\vec{r}$ would be

$$Gm_A\,m_D\;x/r^3 = -(4/27 - 3\sqrt{14}\,/196)Gm_A^{\,2}/d^2,$$

which yields $x = -1.88d$.

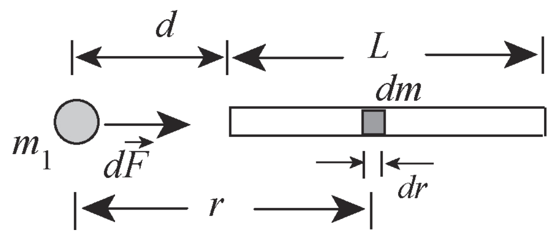
(b) Similarly, $y = -3.90d$,

(c) and $z = 0.489d$.

In this way we are able to deduce that $(x, y, z) = (1.88d, 3.90d, 0.49d)$.

16. Since the rod is an extended object, we cannot apply Equation 13-1 directly to find the force. Instead, we consider a small differential element of the rod, of mass dm of thickness dr at a distance r from m_1. The gravitational force between dm and m_1 is

$$dF = \frac{Gm_1 dm}{r^2} = \frac{Gm_1 (M/L)dr}{r^2},$$



where we have substituted $dm = (M/L)dr$ since mass is uniformly distributed. The direction of $d\vec{F}$ is to the right (see figure). The total force can be found by integrating over the entire length of the rod:

$$F = \int dF = \frac{Gm_1 M}{L}\int_d^{L+d}\frac{dr}{r^2} = -\frac{Gm_1 M}{L}\left(\frac{1}{L+d}-\frac{1}{d}\right) = \frac{Gm_1 M}{d(L+d)}.$$

Substituting the values given in the problem statement, we obtain

$$F = \frac{Gm_1 M}{d(L+d)} = \frac{(6.67\times10^{-11}\ \text{m}^3/\text{kg}\cdot\text{s}^2)(0.67\ \text{kg})(5.0\ \text{kg})}{(0.23\ \text{m})(3.0\ \text{m}+0.23\ \text{m})} = 3.0\times10^{-10}\ \text{N}.$$

17. The acceleration due to gravity is given by $a_g = GM/r^2$, where M is the mass of Earth and r is the distance from Earth's center. We substitute $r = R + h$, where R is the radius of Earth and h is the altitude, to obtain $a_g = GM/(R + h)^2$. We solve for h and obtain $h = \sqrt{GM/a_g} - R$. According to Appendix C, $R = 6.37 \times 10^6$ m and $M = 5.98 \times 10^{24}$ kg, so

$$h = \sqrt{\frac{\left(6.67\times10^{-11}\,\mathrm{m^3/s^2\cdot kg}\right)\left(5.98\times10^{24}\,\mathrm{kg}\right)}{\left(4.9\,\mathrm{m/s^2}\right)}} - 6.37\times10^6\,\mathrm{m} = 2.6\times10^6\,\mathrm{m}.$$

18. We follow the method shown in Sample Problem 13-3. Thus,

$$a_g = \frac{GM_E}{r^2} \Rightarrow da_g = -2\frac{GM_E}{r^3}\,dr$$

which implies that the change in weight is

$$W_{\text{top}} - W_{\text{bottom}} \approx m\left(da_g\right).$$

But since $W_{\text{bottom}} = GmM_E/R^2$ (where R is Earth's mean radius), we have

$$mda_g = -2\frac{GmM_E}{R^3}\,dr = -2W_{\text{bottom}}\frac{dr}{R} = -2\left(600\text{ N}\right)\frac{1.61\times10^3\text{ m}}{6.37\times10^6\text{ m}} = -0.303\text{ N}$$

for the weight change (the minus sign indicating that it is a decrease in W). We are not including any effects due to the Earth's rotation (as treated in Eq. 13-13).

19. (a) The gravitational acceleration at the surface of the Moon is $g_{\text{moon}} = 1.67$ m/s^2 (see Appendix C). The ratio of weights (for a given mass) is the ratio of g-values, so

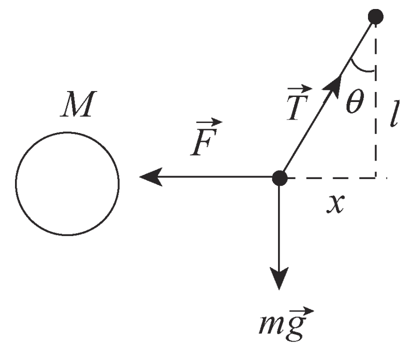
$$W_{\text{moon}} = (100 \text{ N})(1.67/9.8) = 17 \text{ N}.$$

(b) For the force on that object caused by Earth's gravity to equal 17 N, then the free-fall acceleration at its location must be $a_g = 1.67$ m/s^2. Thus,

$$a_g = \frac{Gm_E}{r^2} \Rightarrow r = \sqrt{\frac{Gm_E}{a_g}} = 1.5 \times 10^7 \text{ m}$$

so the object would need to be a distance of $r/R_E = 2.4$ "radii" from Earth's center.

20. The free-body diagram of the force acting on the plumb line is shown on the right. The mass of the sphere is



$$M = \rho V = \rho\left(\frac{4\pi}{3}R^3\right) = \frac{4\pi}{3}(2.6\times10^3\ \text{kg/m}^3)(2.00\times10^3\ \text{m})^3$$
$$= 8.71\times10^{13}\ \text{kg}.$$

The force between the "spherical" mountain and the plumb line is $F = GMm/r^2$. Suppose at equilibrium the line makes an angle θ with the vertical and the net force acting on the line is zero. Therefore,

$$0 = \sum F_{\text{net},\,x} = T\sin\theta - F = T\sin\theta - \frac{GMm}{r^2}$$
$$0 = \sum F_{\text{net},\,y} = T\cos - mg$$

The two equations can be combined to give $\tan\theta = \dfrac{F}{mg} = \dfrac{GM}{gr^2}$. The distance the lower end moves toward the sphere is

$$x = l\tan\theta = l\frac{GM}{gr^2} = (0.50\ \text{m})\frac{(6.67\times10^{-11}\ \text{m}^3/\text{kg}\cdot\text{s}^2)(8.71\times10^{13}\ \text{kg})}{(9.8)(3\times2.00\times10^3\ \text{m})^2}.$$
$$= 8.2\times10^{-6}\ \text{m}.$$

21. (a) The gravitational acceleration is $a_g = \dfrac{GM}{R^2} = 7.6 \text{ m/s}^2$.

(b) Note that the total mass is $5M$. Thus, $a_g = \dfrac{G(5M)}{(3R)^2} = 4.2 \text{ m/s}^2$.

22. (a) Plugging $R_h = 2GM_h / c^2$ into the indicated expression, we find

$$a_g = \frac{GM_h}{(1.001R_h)^2} = \frac{GM_h}{(1.001)^2 \left(2GM_h/c^2\right)^2} = \frac{c^4}{(2.002)^2 \, G} \frac{1}{M_h}$$

which yields $a_g = (3.02 \times 10^{43}\ \text{kg·m/s}^2) / M_h$.

(b) Since M_h is in the denominator of the above result, a_g decreases as M_h increases.

(c) With $M_h = (1.55 \times 10^{12})\ (1.99 \times 10^{30}\ \text{kg})$, we obtain $a_g = 9.82\ \text{m/s}^2$.

(d) This part refers specifically to the very large black hole treated in the previous part. With that mass for M in Eq. 13–16, and $r = 2.002GM/c^2$, we obtain

$$da_g = -2 \frac{GM}{\left(2.002GM/c^2\right)^3} dr = -\frac{2c^6}{(2.002)^3 (GM)^2} dr$$

where $dr \to 1.70$ m as in Sample Problem 13-3. This yields (in absolute value) an acceleration difference of $7.30 \times 10^{-15}\ \text{m/s}^2$.

(e) The miniscule result of the previous part implies that, in this case, any effects due to the differences of gravitational forces on the body are negligible.

23. From Eq. 13-14, we see the extreme case is when "*g*" becomes zero, and plugging in Eq. 13-15 leads to

$$0 = \frac{GM}{R^2} - R\omega^2 \Rightarrow M = \frac{R^3\omega^2}{G}.$$

Thus, with R = 20000 m and ω = 2π rad/s, we find $M = 4.7 \times 10^{24}$ kg ≈ 5×10^{24} kg.

24. (a) What contributes to the GmM/r^2 force on m is the (spherically distributed) mass M contained within r (where r is measured from the center of M). At point A we see that $M_1 + M_2$ is at a smaller radius than $r = a$ and thus contributes to the force:

$$\left|F_{\text{on } m}\right| = \frac{G(M_1 + M_2)m}{a^2}.$$

(b) In the case $r = b$, only M_1 is contained within that radius, so the force on m becomes GM_1m/b^2.

(c) If the particle is at C, then no other mass is at smaller radius and the gravitational force on it is zero.

25. (a) The magnitude of the force on a particle with mass m at the surface of Earth is given by $F = GMm/R^2$, where M is the total mass of Earth and R is Earth's radius. The acceleration due to gravity is

$$a_g = \frac{F}{m} = \frac{GM}{R^2} = \frac{\left(6.67\times10^{-11}\ \text{m}^3/\text{s}^2\cdot\text{kg}\right)\left(5.98\times10^{24}\ \text{kg}\right)}{\left(6.37\times10^{6}\ \text{m}\right)^2} = 9.83\ \text{m/s}^2.$$

(b) Now $a_g = GM/R^2$, where M is the total mass contained in the core and mantle together and R is the outer radius of the mantle (6.345×10^6 m, according to Fig. 13-43). The total mass is

$$M = (1.93 \times 10^{24}\ \text{kg} + 4.01 \times 10^{24}\ \text{kg}) = 5.94 \times 10^{24}\ \text{kg}.$$

The first term is the mass of the core and the second is the mass of the mantle. Thus,

$$a_g = \frac{\left(6.67\times10^{-11}\ \text{m}^3/\text{s}^2\cdot\text{kg}\right)\left(5.94\times10^{24}\ \text{kg}\right)}{\left(6.345\times10^{6}\ \text{m}\right)^2} = 9.84\ \text{m/s}^2.$$

(c) A point 25 km below the surface is at the mantle-crust interface and is on the surface of a sphere with a radius of $R = 6.345 \times 10^6$ m. Since the mass is now assumed to be uniformly distributed the mass within this sphere can be found by multiplying the mass per unit volume by the volume of the sphere: $M = (R^3/R_e^3)M_e$, where M_e is the total mass of Earth and R_e is the radius of Earth. Thus,

$$M = \left(\frac{6.345\times10^{6}\ \text{m}}{6.37\times10^{6}\ \text{m}}\right)^3\left(5.98\times10^{24}\ \text{kg}\right) = 5.91\times10^{24}\ \text{kg}.$$

The acceleration due to gravity is

$$a_g = \frac{GM}{R^2} = \frac{\left(6.67\times10^{-11}\ \text{m}^3/\text{s}^2\cdot\text{kg}\right)\left(5.91\times10^{24}\ \text{kg}\right)}{\left(6.345\times10^{6}\ \text{m}\right)^2} = 9.79\ \text{m/s}^2.$$

26. (a) Using Eq. 13-1, we set GmM/r^2 equal to $\frac{1}{2}GmM/R^2$, and we find $r = R\sqrt{2}$. Thus, the distance from the surface is $(\sqrt{2}-1)R = 0.414R$.

(b) Setting the density ρ equal to M/V where $V = \frac{4}{3}\pi R^3$, we use Eq. 13-19:

$$F = \frac{4\pi Gmr\rho}{3} = \frac{4\pi Gmr}{3}\left(\frac{M}{4\pi R^3/3}\right) = \frac{GMmr}{R^3} = \frac{1}{2}\frac{GMm}{R^2} \quad \Rightarrow \quad r = R/2.$$

27. Using the fact that the volume of a sphere is $4\pi R^3/3$, we find the density of the sphere:

$$\rho = \frac{M_{\text{total}}}{\frac{4}{3}\pi R^3} = \frac{1.0\times10^4\,\text{kg}}{\frac{4}{3}\pi\left(1.0\text{ m}\right)^3} = 2.4\times10^3\,\text{kg/m}^3.$$

When the particle of mass m (upon which the sphere, or parts of it, are exerting a gravitational force) is at radius r (measured from the center of the sphere), then whatever mass M is at a radius less than r must contribute to the magnitude of that force (GMm/r^2).

(a) At $r = 1.5$ m, all of M_{total} is at a smaller radius and thus all contributes to the force:

$$\left|F_{\text{on } m}\right| = \frac{GmM_{\text{total}}}{r^2} = m\left(3.0\times10^{-7}\,\text{N/kg}\right).$$

(b) At $r = 0.50$ m, the portion of the sphere at radius smaller than that is

$$M = \rho\left(\frac{4}{3}\pi r^3\right) = 1.3\times10^3\ \text{kg}.$$

Thus, the force on m has magnitude $GMm/r^2 = m\ (3.3 \times 10^{-7}$ N/kg).

(c) Pursuing the calculation of part (b) algebraically, we find

$$\left|F_{\text{on } m}\right| = \frac{Gm\rho\left(\frac{4}{3}\pi r^3\right)}{r^2} = mr\left(6.7\times10^{-7}\,\frac{\text{N}}{\text{kg}\cdot\text{m}}\right).$$

28. The difference between free-fall acceleration g and the gravitational acceleration a_g at the equator of the star is (see Equation 13.14):

$$a_g - g = \omega^2 R$$

where

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{0.041\,\mathrm{s}} = 153\,\mathrm{rad/s}$$

is the angular speed of the star. The gravitational acceleration at the equator is

$$a_g = \frac{GM}{R^2} = \frac{(6.67\times10^{-11}\ \mathrm{m^3/kg\cdot s^2})(1.98\times10^{30}\ \mathrm{kg})}{(1.2\times10^4\ \mathrm{m})^2} = 9.17\times10^{11}\ \mathrm{m/s^2}.$$

Therefore, the percentage difference is

$$\frac{a_g - g}{a_g} = \frac{\omega^2 R}{a_g} = \frac{(153\,\mathrm{rad/s})^2(1.2\times10^4\ \mathrm{m})}{9.17\times10^{11}\ \mathrm{m/s^2}} = 3.06\times10^{-4} \approx 0.031\%.$$

29. (a) The density of a uniform sphere is given by $\rho = 3M/4\pi R^3$, where M is its mass and R is its radius. The ratio of the density of Mars to the density of Earth is

$$\frac{\rho_M}{\rho_E} = \frac{M_M}{M_E}\frac{R_E^3}{R_M^3} = 0.11\left(\frac{0.65\times10^4\ \text{km}}{3.45\times10^3\ \text{km}}\right)^3 = 0.74.$$

(b) The value of a_g at the surface of a planet is given by $a_g = GM/R^2$, so the value for Mars is

$$a_gM = \frac{M_M}{M_E}\frac{R_E^2}{R_M^2}a_{gE} = 0.11\left(\frac{0.65\times10^4\ \text{km}}{3.45\times10^3\ \text{km}}\right)^2\left(9.8\ \text{m/s}^2\right) = 3.8\ \text{m/s}^2.$$

(c) If v is the escape speed, then, for a particle of mass m

$$\frac{1}{2}mv^2 = G\frac{mM}{R} \quad \Rightarrow \quad v = \sqrt{\frac{2GM}{R}}.$$

For Mars, the escape speed is

$$v = \sqrt{\frac{2(6.67\times10^{-11}\ \text{m}^3/\text{s}^2\cdot\text{kg})(0.11)\left(5.98\times10^{24}\ \text{kg}\right)}{3.45\times10^6\ \text{m}}} = 5.0\times10^3\ \text{m/s}.$$

30. (a) The gravitational potential energy is

$$U = -\frac{GMm}{r} = -\frac{\left(6.67\times10^{-11}\ \text{m}^3/\text{s}^2\cdot\text{kg}\right)\left(5.2\ \text{kg}\right)\left(2.4\ \text{kg}\right)}{19\ \text{m}} = -\ 4.4\times10^{-11}\ \text{J}.$$

(b) Since the change in potential energy is

$$\Delta U = -\frac{GMm}{3r} - \left(-\frac{GMm}{r}\right) = -\frac{2}{3}\left(-4.4\times10^{-11}\ \text{J}\right) = 2.9\times10^{-11}\ \text{J},$$

the work done by the gravitational force is $W = -\ \Delta U = -2.9 \times 10^{-11}$ J.

(c) The work done by you is $W' = \Delta U = 2.9 \times 10^{-11}$ J.

31. The amount of (kinetic) energy needed to escape is the same as the (absolute value of the) gravitational potential energy at its original position. Thus, an object of mass m on a planet of mass M and radius R needs $K = GmM/R$ in order to (barely) escape.
(a) Setting up the ratio, we find

$$\frac{K_m}{K_E} = \frac{M_m}{M_E}\frac{R_E}{R_m} = 0.0451$$

using the values found in Appendix C.

(b) Similarly, for the Jupiter escape energy (divided by that for Earth) we obtain

$$\frac{K_J}{K_E} = \frac{M_J}{M_E}\frac{R_E}{R_J} = 28.5.$$

32. (a) The potential energy at the surface is (according to the graph) -5.0×10^9 J, so (since U is inversely proportional to r – see Eq. 13-21) at an r-value a factor of 5/4 times what it was at the surface then U must be a factor of 4/5 what it was. Thus, at $r = 1.25R_s$ $U = -4.0 \times 10^9$ J. Since mechanical energy is assumed to be conserved in this problem, we have $K + U = -2.0 \times 10^9$ J at this point. Since $U = -4.0 \times 10^9$ J here, then $K = 2.0 \times 10^9 \text{ J}$ at this point.

(b) To reach the point where the mechanical energy equals the potential energy (that is, where $U = -2.0 \times 10^9$ J) means that U must reduce (from its value at $r = 1.25R_s$) by a factor of 2 – which means the r value must increase (relative to $r = 1.25R_s$) by a corresponding factor of 2. Thus, the turning point must be at $r = 2.5R_s$.

33. The equation immediately preceding Eq. 13-28 shows that $K = -U$ (with U evaluated at the planet's surface: -5.0×10^9 J) is required to "escape." Thus, $K = 5.0 \times 10^9$ J.

34. The gravitational potential energy is

$$U = -\frac{Gm(M-m)}{r} = -\frac{G}{r}\left(Mm - m^2\right)$$

which we differentiate with respect to m and set equal to zero (in order to minimize). Thus, we find $M - 2m = 0$ which leads to the ratio $m/M = 1/2$ to obtain the least potential energy.

Note that a second derivative of U with respect to m would lead to a positive result regardless of the value of m − which means its graph is everywhere concave upward and thus its extremum is indeed a minimum.

35. (a) The work done by you in moving the sphere of mass m_B equals the change in the potential energy of the three-sphere system. The initial potential energy is

$$U_i = -\frac{Gm_A m_B}{d} - \frac{Gm_A m_C}{L} - \frac{Gm_B m_C}{L-d}$$

and the final potential energy is

$$U_f = -\frac{Gm_A m_B}{L-d} - \frac{Gm_A m_C}{L} - \frac{Gm_B m_C}{d}.$$

The work done is

$$W = U_f - U_i = Gm_B\left[m_A\left(\frac{1}{d} - \frac{1}{L-d}\right) + m_C\left(\frac{1}{L-d} - \frac{1}{d}\right)\right]$$

$$= Gm_B\left[m_A\frac{L-2d}{d(L-d)} + m_C\frac{2d-L}{d(L-d)}\right] = Gm_B(m_A - m_C)\frac{L-2d}{d(L-d)}$$

$$= (6.67\times 10^{-11}\ \text{m}^3/\text{s}^2\cdot\text{kg})(0.010\ \text{kg})(0.080\ \text{kg} - 0.020\ \text{kg})\frac{0.12\ \text{m} - 2(0.040\ \text{m})}{(0.040\ \text{m})(0.12 - 0.040\ \text{m})}$$

$$= +5.0\times 10^{-13}\ \text{J}.$$

(b) The work done by the force of gravity is $-(U_f - U_i) = -5.0\times 10^{-13}$ J.

36. (a) From Eq. 13-28, we see that $v_0 = \sqrt{GM/2R_E}$ in this problem. Using energy conservation, we have

$$\frac{1}{2}mv_o^2 - GMm/R_E = -GMm/r$$

which yields $r = 4R_E/3$. So the multiple of R_E is 4/3 or 1.33.

(b) Using the equation in the textbook immediately preceding Eq. 13-28, we see that in this problem we have $K_i = GMm/2R_E$, and the above manipulation (using energy conservation) in this case leads to $r = 2R_E$. So the multiple of R_E is 2.00.

(c) Again referring to the equation in the textbook immediately preceding Eq. 13-28, we see that the mechanical energy = 0 for the "escape condition."

37. (a) We use the principle of conservation of energy. Initially the particle is at the surface of the asteroid and has potential energy $U_i = -GMm/R$, where M is the mass of the asteroid, R is its radius, and m is the mass of the particle being fired upward. The initial kinetic energy is $\frac{1}{2}mv^2$. The particle just escapes if its kinetic energy is zero when it is infinitely far from the asteroid. The final potential and kinetic energies are both zero. Conservation of energy yields

$$-GMm/R + \tfrac{1}{2}mv^2 = 0.$$

We replace GM/R with a_gR, where a_g is the acceleration due to gravity at the surface. Then, the energy equation becomes $-a_gR + \frac{1}{2}v^2 = 0$. We solve for v:

$$v = \sqrt{2a_gR} = \sqrt{2(3.0\ \text{m/s}^2)(500\times10^3\ \text{m})} = 1.7\times10^3\ \text{m/s}.$$

(b) Initially the particle is at the surface; the potential energy is $U_i = -GMm/R$ and the kinetic energy is $K_i = \frac{1}{2}mv^2$. Suppose the particle is a distance h above the surface when it momentarily comes to rest. The final potential energy is $U_f = -GMm/(R + h)$ and the final kinetic energy is $K_f = 0$. Conservation of energy yields

$$-\frac{GMm}{R} + \frac{1}{2}mv^2 = -\frac{GMm}{R+h}.$$

We replace GM with a_gR^2 and cancel m in the energy equation to obtain

$$-a_gR + \frac{1}{2}v^2 = -\frac{a_gR^2}{(R+h)}.$$

The solution for h is

$$\begin{aligned} h &= \frac{2a_gR^2}{2a_gR - v^2} - R = \frac{2(3.0\ \text{m/s}^2)(500\times10^3\ \text{m})^2}{2(3.0\ \text{m/s}^2)(500\times10^3\ \text{m}) - (1000\ \text{m/s})^2} - (500\times10^3\ \text{m}) \\ &= 2.5\times10^5\ \text{m}. \end{aligned}$$

(c) Initially the particle is a distance h above the surface and is at rest. Its potential energy is $U_i = -GMm/(R + h)$ and its initial kinetic energy is $K_i = 0$. Just before it hits the asteroid its potential energy is $U_f = -GMm/R$. Write $\frac{1}{2}mv_f^2$ for the final kinetic energy. Conservation of energy yields

$$-\frac{GMm}{R+h} = -\frac{GMm}{R} + \frac{1}{2}mv^2.$$

We substitute $a_g R^2$ for GM and cancel m, obtaining

$$-\frac{a_g R^2}{R+h} = -a_g R + \frac{1}{2}v^2.$$

The solution for v is

$$v = \sqrt{2a_g R - \frac{2a_g R^2}{R+h}} = \sqrt{2(3.0\ \text{m/s}^2)(500\times10^3\ \text{m}) - \frac{2(3.0\ \text{m/s}^2)(500\times10^3\ \text{m})^2}{(500\times10^3\ \text{m})+(1000\times10^3\ \text{m})}}$$
$$= 1.4\times10^3\ \text{m/s}.$$

38. Energy conservation for this situation may be expressed as follows:

$$K_1 + U_1 = K_2 + U_2 \quad \Rightarrow \quad K_1 - \frac{GmM}{r_1} = K_2 - \frac{GmM}{r_2}.$$

where $M = 5.0 \times 10^{23}$ kg, $r_1 = R = 3.0 \times 10^6$ m and $m = 10$ kg.

(a) If $K_1 = 5.0 \times 10^7$ J and $r_2 = 4.0 \times 10^6$ m, then the above equation leads to

$$K_2 = K_1 + GmM\left(\frac{1}{r_2} - \frac{1}{r_1}\right) = 2.2 \times 10^7 \text{ J}.$$

(b) In this case, we require $K_2 = 0$ and $r_2 = 8.0 \times 10^6$ m, and solve for K_1:

$$K_1 = K_2 + GmM\left(\frac{1}{r_1} - \frac{1}{r_2}\right) = 6.9 \times 10^7 \text{ J}.$$

39. (a) The momentum of the two-star system is conserved, and since the stars have the same mass, their speeds and kinetic energies are the same. We use the principle of conservation of energy. The initial potential energy is $U_i = -GM^2/r_i$, where M is the mass of either star and r_i is their initial center-to-center separation. The initial kinetic energy is zero since the stars are at rest. The final potential energy is $U_f = -2GM^2/r_i$ since the final separation is $r_i/2$. We write Mv^2 for the final kinetic energy of the system. This is the sum of two terms, each of which is $\frac{1}{2}Mv^2$. Conservation of energy yields

$$-\frac{GM^2}{r_i} = -\frac{2GM^2}{r_i} + Mv^2.$$

The solution for v is

$$v = \sqrt{\frac{GM}{r_i}} = \sqrt{\frac{(6.67 \times 10^{-11}\ \text{m}^3/\text{s}^2 \cdot \text{kg})(10^{30}\ \text{kg})}{10^{10}\ \text{m}}} = 8.2 \times 10^4\ \text{m/s}.$$

(b) Now the final separation of the centers is $r_f = 2R = 2 \times 10^5$ m, where R is the radius of either of the stars. The final potential energy is given by $U_f = -GM^2/r_f$ and the energy equation becomes $-GM^2/r_i = -GM^2/r_f + Mv^2$. The solution for v is

$$v = \sqrt{GM\left(\frac{1}{r_f} - \frac{1}{r_i}\right)} = \sqrt{(6.67 \times 10^{-11}\text{m}^3/\text{s}^2 \cdot \text{kg})(10^{30}\ \text{kg})\left(\frac{1}{2 \times 10^5\ \text{m}} - \frac{1}{10^{10}\ \text{m}}\right)}$$
$$= 1.8 \times 10^7\ \text{m/s}.$$

40. (a) The initial gravitational potential energy is

$$U_i = -\frac{GM_AM_B}{r_i} = -\frac{(6.67\times10^{-11}\ \text{m}^3/\text{s}^2\cdot\text{kg})(20\ \text{kg})(10\ \text{kg})}{0.80\ \text{m}}$$
$$= -1.67\times10^{-8}\ \text{J} \approx -1.7\times10^{-8}\ \text{J}.$$

(b) We use conservation of energy (with $K_i = 0$):

$$U_i = K + U \quad \Rightarrow \quad -1.7\times10^{-8} = K - \frac{(6.67\times10^{-11}\ \text{m}^3/\text{s}^2\cdot\text{kg})(20\ \text{kg})(10\ \text{kg})}{0.60\ \text{m}}$$

which yields $K = 5.6 \times 10^{-9}$ J. Note that the value of r is the difference between 0.80 m and 0.20 m.

41. Let m = 0.020 kg and d = 0.600 m (the original edge-length, in terms of which the final edge-length is d/3). The total initial gravitational potential energy (using Eq. 13-21 and some elementary trigonometry) is

$$U_i = -\frac{4Gm^2}{d} - \frac{2Gm^2}{\sqrt{2}\,d} \ .$$

Since U is inversely proportional to r then reducing the size by 1/3 means increasing the magnitude of the potential energy by a factor of 3, so

$$U_f = 3U_i \ \Rightarrow \ \Delta U = 2U_i = 2(4+\sqrt{2}\,)\left(-\frac{Gm^2}{d}\right) = -4.82\times10^{-13}\ \text{J} \ .$$

42. (a) Applying Eq. 13-21 and the Pythagorean theorem leads to

$$U = -\left(\frac{GM^2}{2D} + \frac{2GmM}{\sqrt{y^2 + D^2}}\right)$$

where M is the mass of particle B (also that of particle C) and m is the mass of particle A. The value given in the problem statement (for infinitely large y, for which the second term above vanishes) determines M, since D is given. Thus $M = 0.50$ kg.

(b) We estimate (from the graph) the $y = 0$ value to be $U_o = -3.5 \times 10^{-10}$ J. Using this, our expression above determines m. We obtain $m = 1.5$ kg.

43. The period T and orbit radius r are related by the law of periods: $T^2 = (4\pi^2/GM)r^3$, where M is the mass of Mars. The period is 7 h 39 min, which is 2.754×10^4 s. We solve for M:

$$M = \frac{4\pi^2 r^3}{GT^2} = \frac{4\pi^2 (9.4 \times 10^6\ \text{m})^3}{(6.67 \times 10^{-11}\,\text{m}^3/\text{s}^2 \cdot \text{kg})\left(2.754 \times 10^4\ \text{s}\right)^2} = 6.5 \times 10^{23}\ \text{kg}.$$

44. From Eq. 13-37, we obtain $v = \sqrt{GM/r}$ for the speed of an object in circular orbit (of radius r) around a planet of mass M. In this case, $M = 5.98 \times 10^{24}$ kg and

$$r = (700 + 6370)\text{m} = 7070 \text{ km} = 7.07 \times 10^{6} \text{ m}.$$

The speed is found to be $v = 7.51 \times 10^{3}$ m/s. After multiplying by 3600 s/h and dividing by 1000 m/km this becomes $v = 2.7 \times 10^{4}$ km/h.

(a) For a head-on collision, the relative speed of the two objects must be $2v = 5.4 \times 10^{4}$ km/h.

(b) A perpendicular collision is possible if one satellite is, say, orbiting above the equator and the other is following a longitudinal line. In this case, the relative speed is given by the Pythagorean theorem: $\sqrt{v^2 + v^2} = 3.8 \times 10^{4}$ km/h.

45. Let N be the number of stars in the galaxy, M be the mass of the Sun, and r be the radius of the galaxy. The total mass in the galaxy is $N\,M$ and the magnitude of the gravitational force acting on the Sun is $F = GNM^2/r^2$. The force points toward the galactic center. The magnitude of the Sun's acceleration is $a = v^2/R$, where v is its speed. If T is the period of the Sun's motion around the galactic center then $v = 2\pi R/T$ and $a = 4\pi^2 R/T^2$. Newton's second law yields $GNM^2/R^2 = 4\pi^2 MR/T^2$. The solution for N is

$$N = \frac{4\pi^2 R^3}{GT^2 M}.$$

The period is 2.5×10^8 y, which is 7.88×10^{15} s, so

$$N = \frac{4\pi^2\,(2.2\times10^{20}\ \text{m})^3}{(6.67\times10^{-11}\ \text{m}^3/\text{s}^2\cdot\text{kg})\,(7.88\times10^{15}\ \text{s})^2\,(2.0\times10^{30}\ \text{kg})} = 5.1\times10^{10}.$$

46. Kepler’s law of periods, expressed as a ratio, is

$$\left(\frac{a_M}{a_E}\right)^3 = \left(\frac{T_M}{T_E}\right)^2 \Rightarrow (1.52)^3 = \left(\frac{T_M}{1\,\text{y}}\right)^2$$

where we have substituted the mean-distance (from Sun) ratio for the semi-major axis ratio. This yields T_M = 1.87 y. The value in Appendix C (1.88 y) is quite close, and the small apparent discrepancy is not significant, since a more precise value for the semi-major axis ratio is a_M/a_E = 1.523 which does lead to T_M = 1.88 y using Kepler’s law. A question can be raised regarding the use of a ratio of mean distances for the ratio of semi-major axes, but this requires a more lengthy discussion of what is meant by a ”mean distance” than is appropriate here.

47. (a) The greatest distance between the satellite and Earth's center (the apogee distance) and the least distance (perigee distance) are, respectively,

$$R_a = (6.37 \times 10^6 \text{ m} + 360 \times 10^3 \text{ m}) = 6.73 \times 10^6 \text{ m}$$
$$R_p = (6.37 \times 10^6 \text{ m} + 180 \times 10^3 \text{ m}) = 6.55 \times 10^6 \text{ m}.$$

Here 6.37×10^6 m is the radius of Earth. From Fig. 13-13, we see that the semi-major axis is

$$a = \frac{R_a + R_p}{2} = \frac{6.73 \times 10^6 \text{ m} + 6.55 \times 10^6 \text{ m}}{2} = 6.64 \times 10^6 \text{ m}.$$

(b) The apogee and perigee distances are related to the eccentricity e by $R_a = a(1 + e)$ and $R_p = a(1 - e)$. Add to obtain $R_a + R_p = 2a$ and $a = (R_a + R_p)/2$. Subtract to obtain $R_a - R_p = 2ae$. Thus,

$$e = \frac{R_a - R_p}{2a} = \frac{R_a - R_p}{R_a + R_p} = \frac{6.73 \times 10^6 \text{ m} - 6.55 \times 10^6 \text{ m}}{6.73 \times 10^6 \text{ m} + 6.55 \times 10^6 \text{ m}} = 0.0136.$$

48. Kepler's law of periods, expressed as a ratio, is

$$\left(\frac{r_s}{r_m}\right)^3 = \left(\frac{T_s}{T_m}\right)^2 \Rightarrow \left(\frac{1}{2}\right)^3 = \left(\frac{T_s}{1 \text{ lunar month}}\right)^2$$

which yields $T_s = 0.35$ lunar month for the period of the satellite.

49. (a) If r is the radius of the orbit then the magnitude of the gravitational force acting on the satellite is given by GMm/r^2, where M is the mass of Earth and m is the mass of the satellite. The magnitude of the acceleration of the satellite is given by v^2/r, where v is its speed. Newton's second law yields $GMm/r^2 = mv^2/r$. Since the radius of Earth is 6.37×10^6 m the orbit radius is $r = (6.37 \times 10^6 \text{ m} + 160 \times 10^3 \text{ m}) = 6.53 \times 10^6$ m. The solution for v is

$$v = \sqrt{\frac{GM}{r}} = \sqrt{\frac{(6.67 \times 10^{-11} \text{ m}^3/\text{s}^2 \cdot \text{kg})(5.98 \times 10^{24} \text{ kg})}{6.53 \times 10^6 \text{ m}}} = 7.82 \times 10^3 \text{ m/s}.$$

(b) Since the circumference of the circular orbit is $2\pi r$, the period is

$$T = \frac{2\pi r}{v} = \frac{2\pi(6.53 \times 10^6 \text{ m})}{7.82 \times 10^3 \text{ m/s}} = 5.25 \times 10^3 \text{ s}.$$

This is equivalent to 87.5 min.

50. (a) The distance from the center of an ellipse to a focus is *ae* where *a* is the semimajor axis and *e* is the eccentricity. Thus, the separation of the foci (in the case of Earth's orbit) is

$$2ae = 2\left(1.50 \times 10^{11}\ \text{m}\right)\left(0.0167\right) = 5.01 \times 10^{9}\ \text{m}.$$

(b) To express this in terms of solar radii (see Appendix C), we set up a ratio:

$$\frac{5.01 \times 10^{9}\ \text{m}}{6.96 \times 10^{8}\ \text{m}} = 7.20.$$

51. (a) The period of the comet is 1420 years (and one month), which we convert to T = 4.48×10^{10} s. Since the mass of the Sun is 1.99×10^{30} kg, then Kepler's law of periods gives

$$(4.48 \times 10^{10}\ \text{s})^2 = \left(\frac{4\pi^2}{(6.67\times10^{-11}\ \text{m}^3/\text{kg}\cdot\text{s}^2)(1.99\times10^{30}\ \text{kg})}\right) a^3 \Rightarrow a = 1.89\times10^{13}\ \text{m}.$$

(b) Since the distance from the focus (of an ellipse) to its center is ea and the distance from center to the aphelion is a, then the comet is at a distance of

$$ea + a = (0.11+1)\ (1.89\times10^{13}\ \text{m}) = 2.1\times10^{13}\ \text{m}$$

when it is farthest from the Sun. To express this in terms of Pluto's orbital radius (found in Appendix C), we set up a ratio:

$$\left(\frac{2.1\times10^{13}}{5.9\times10^{12}}\right) R_P = 3.6 R_P.$$

52. To “hover” above Earth ($M_E = 5.98 \times 10^{24}$ kg) means that it has a period of 24 hours (86400 s). By Kepler’s law of periods,

$$(86400)^2 = \left(\frac{4\pi^2}{GM_E}\right) r^3 \Rightarrow r = 4.225 \times 10^7 \text{ m.}$$

Its altitude is therefore $r - R_E$ (where $R_E = 6.37 \times 10^6$ m) which yields 3.58×10^7 m.

53. (a) If we take the logarithm of Kepler's law of periods, we obtain

$$2\log(T) = \log(4\pi^2/GM) + 3\log(a) \Rightarrow \log(a) = \frac{2}{3}\log(T) - \frac{1}{3}\log(4\pi^2/GM)$$

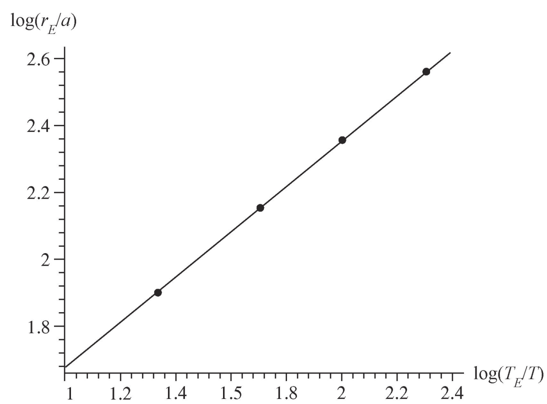
where we are ignoring an important subtlety about units (the arguments of logarithms cannot have units, since they are transcendental functions). Although the problem can be continued in this way, we prefer to set it up without units, which requires taking a ratio. If we divide Kepler's law (applied to the Jupiter-moon system, where M is mass of Jupiter) by the law applied to Earth orbiting the Sun (of mass M_o), we obtain

$$(T/T_E)^2 = \left(\frac{M_o}{M}\right)\left(\frac{a}{r_E}\right)^3$$

where T_E = 365.25 days is Earth's orbital period and $r_E = 1.50 \times 10^{11}$ m is its mean distance from the Sun. In this case, it is perfectly legitimate to take logarithms and obtain

$$\log\left(\frac{r_E}{a}\right) = \frac{2}{3}\log\left(\frac{T_E}{T}\right) + \frac{1}{3}\log\left(\frac{M_o}{M}\right)$$

(written to make each term positive) which is the way we plot the data ($\log(r_E/a)$ on the vertical axis and $\log(T_E/T)$ on the horizontal axis).



(b) When we perform a least-squares fit to the data, we obtain

$$\log(r_E/a) = 0.666\log(T_E/T) + 1.01,$$

which confirms the expectation of slope = 2/3 based on the above equation.

(c) And the 1.01 intercept corresponds to the term 1/3 $\log(M_o/M)$ which implies

$$\frac{M_o}{M} = 10^{3.03} \Rightarrow M = \frac{M_o}{1.07\times 10^3}.$$

Plugging in $M_o = 1.99 \times 10^{30}$ kg (see Appendix C), we obtain $M = 1.86 \times 10^{27}$ kg for Jupiter's mass. This is reasonably consistent with the value 1.90×10^{27} kg found in Appendix C.

54. (a) The period is $T = 27(3600) = 97200$ s, and we are asked to assume that the orbit is circular (of radius r = 100000 m). Kepler's law of periods provides us with an approximation to the asteroid's mass:

$$(97200)^2 = \left(\frac{4\pi^2}{GM}\right)(100000)^3 \Rightarrow M = 6.3\times 10^{16} \text{ kg.}$$

(b) Dividing the mass M by the given volume yields an average density equal to

$$\rho = 6.3\times 10^{16}/1.41\times 10^{13} = 4.4\times 10^{3} \text{ kg/m}^3,$$

which is about 20% less dense than Earth.

55. In our system, we have $m_1 = m_2 = M$ (the mass of our Sun, 1.99×10^{30} kg). With $r = 2r_1$ in this system (so r_1 is one-half the Earth-to-Sun distance r), and $v = \pi r/T$ for the speed, we have

$$\frac{Gm_1m_2}{r^2} = m_1 \frac{(\pi r/T)^2}{r/2} \Rightarrow T = \sqrt{\frac{2\pi^2 r^3}{GM}}.$$

With $r = 1.5 \times 10^{11}$ m, we obtain $T = 2.2 \times 10^7$ s. We can express this in terms of Earth-years, by setting up a ratio:

$$T = \left(\frac{T}{1\,\text{y}}\right)(1\,\text{y}) = \left(\frac{2.2 \times 10^7\ \text{s}}{3.156 \times 10^7\ \text{s}}\right)(1\ \text{y}) = 0.71\ \text{y}.$$

56. The two stars are in circular orbits, not about each other, but about the two-star system's center of mass (denoted as O), which lies along the line connecting the centers of the two stars. The gravitational force between the stars provides the centripetal force necessary to keep their orbits circular. Thus, for the visible, Newton's second law gives

$$F = \frac{Gm_1m_2}{r^2} = \frac{m_1v^2}{r_1}$$

where r is the distance between the centers of the stars. To find the relation between r and r_1, we locate the center of mass relative to m_1. Using Equation 9-1, we obtain

$$r_1 = \frac{m_1(0)+m_2r}{m_1+m_2} = \frac{m_2r}{m_1+m_2} \quad \Rightarrow \quad r = \frac{m_1+m_2}{m_2}r_1 .$$

On the other hand, since the orbital speed of m_1 is $v = 2\pi r_1 / T$, then $r_1 = vT/2\pi$ and the expression for r can be rewritten as

$$r = \frac{m_1+m_2}{m_2}\frac{vT}{2\pi} .$$

Substituting r and r_1 into the force equation, we obtain

$$F = \frac{4\pi^2Gm_1m_2^3}{(m_1+m_2)^2v^2T^2} = \frac{2\pi m_1v}{T}$$

or

$$\frac{m_2^3}{(m_1+m_2)^2} = \frac{v^3T}{2\pi G} = \frac{(2.7\times10^5 \text{ m/s})^3(1.70 \text{ days})(86400 \text{ s/day})}{2\pi(6.67\times10^{-11} \text{ m}^3/\text{kg}\cdot\text{s}^2)} = 6.90\times10^{30} \text{ kg}$$
$$= 3.467M_s ,$$

where $M_s = 1.99\times10^{30}$ kg is the mass of the sun. With $m_1 = 6M_s$, we write $m_2 = \alpha M_s$ and solve the following cubic equation for α:

$$\frac{\alpha^3}{(6+\alpha)^2} - 3.467 = 0 .$$

The equation has one real solution: $\alpha = 9.3$, which implies $m_2 / M_s \approx 9$.

57. From Kepler's law of periods (where T = 2.4(3600) = 8640 s), we find the planet's mass M:

$$(8640\,\text{s})^2 = \left(\frac{4\pi^2}{GM}\right)(8.0\times10^6\ \text{m})^3 \Rightarrow M = 4.06\times10^{24}\ \text{kg}.$$

But we also know $a_g = GM/R^2 = 8.0\ \text{m/s}^2$ so that we are able to solve for the planet's radius:

$$R = \sqrt{\frac{GM}{a_g}} = 5.8\times10^6\ \text{m}.$$

58. (a) We make use of

$$\frac{m_2^3}{(m_1 + m_2)^2} = \frac{v^3 T}{2\pi G}$$

where $m_1 = 0.9M_{\text{Sun}}$ is the estimated mass of the star. With $v = 70$ m/s and $T = 1500$ days (or $1500 \times 86400 = 1.3 \times 10^8$ s), we find

$$\frac{m_2^3}{(0.9M_{\text{Sun}} + m_2)^2} = 1.06 \times 10^{23} \text{ kg} .$$

Since $M_{\text{Sun}} \approx 2.0 \times 10^{30}$ kg, we find $m_2 \approx 7.0 \times 10^{27}$ kg. Dividing by the mass of Jupiter (see Appendix C), we obtain $m \approx 3.7 m_J$.

(b) Since $v = 2\pi r_1/T$ is the speed of the star, we find

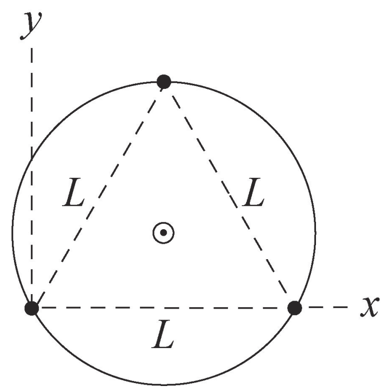
$$r_1 = \frac{vT}{2\pi} = \frac{(70\,\text{m/s})(1.3 \times 10^8\,\text{s})}{2\pi} = 1.4 \times 10^9 \text{ m}$$

for the star's orbital radius. If r is the distance between the star and the planet, then $r_2 = r - r_1$ is the orbital radius of the planet, and is given by

$$r_2 = r_1\left(\frac{m_1 + m_2}{m_2} - 1\right) = r_1\frac{m_1}{m_2} = 3.7 \times 10^{11} \text{ m} .$$

Dividing this by 1.5×10^{11} m (Earth's orbital radius, r_E) gives $r_2 = 2.5 r_E$.

59. Each star is attracted toward each of the other two by a force of magnitude GM^2/L^2, along the line that joins the stars. The net force on each star has magnitude $2(GM^2/L^2)\cos 30°$ and is directed toward the center of the triangle. This is a centripetal force and keeps the stars on the same circular orbit if their speeds are appropriate. If R is the radius of the orbit, Newton's second law yields $(GM^2/L^2)\cos 30° = Mv^2/R$.



The stars rotate about their center of mass (marked by a circled dot on the diagram above) at the intersection of the perpendicular bisectors of the triangle sides, and the radius of the orbit is the distance from a star to the center of mass of the three-star system. We take the coordinate system to be as shown in the diagram, with its origin at the left-most star. The altitude of an equilateral triangle is $\left(\sqrt{3}/2\right)L$, so the stars are located at $x = 0$, $y = 0$; $x = L$, $y = 0$; and $x = L/2$, $y = \sqrt{3}L/2$. The x coordinate of the center of mass is $x_c = (L + L/2)/3 = L/2$ and the y coordinate is $y_c = \left(\sqrt{3}L/2\right)/3 = L/2\sqrt{3}$. The distance from a star to the center of mass is

$$R = \sqrt{x_c^2 + y_c^2} = \sqrt{\left(L^2/4\right) + \left(L^2/12\right)} = L/\sqrt{3}.$$

Once the substitution for R is made Newton's second law becomes $\left(2GM^2/L^2\right)\cos 30° = \sqrt{3}Mv^2/L$. This can be simplified somewhat by recognizing that $\cos 30° = \sqrt{3}/2$, and we divide the equation by M. Then, $GM/L^2 = v^2/L$ and $v = \sqrt{GM/L}$.

60. Although altitudes are given, it is the orbital radii which enter the equations. Thus, r_A = (6370 + 6370) km = 12740 km, and r_B = (19110 + 6370) km = 25480 km

(a) The ratio of potential energies is

$$\frac{U_B}{U_A} = \frac{-GmM / r_B}{-GmM / r_A} = \frac{r_A}{r_B} = \frac{1}{2}.$$

(b) Using Eq. 13-38, the ratio of kinetic energies is

$$\frac{K_B}{K_A} = \frac{GmM / 2r_B}{GmM / 2r_A} = \frac{r_A}{r_B} = \frac{1}{2}.$$

(c) From Eq. 13-40, it is clear that the satellite with the largest value of r has the smallest value of $|E|$ (since r is in the denominator). And since the values of E are negative, then the smallest value of $|E|$ corresponds to the largest energy E. Thus, satellite B has the largest energy.

(d) The difference is

$$\Delta E = E_B - E_A = -\frac{GmM}{2}\left(\frac{1}{r_B} - \frac{1}{r_A}\right).$$

Being careful to convert the r values to meters, we obtain $\Delta E = 1.1 \times 10^8$ J. The mass M of Earth is found in Appendix C.

61. (a) We use the law of periods: $T^2 = (4\pi^2/GM)r^3$, where M is the mass of the Sun (1.99×10^{30} kg) and r is the radius of the orbit. The radius of the orbit is twice the radius of Earth's orbit: $r = 2r_e = 2(150 \times 10^9 \text{ m}) = 300 \times 10^9$ m. Thus,

$$T = \sqrt{\frac{4\pi^2 r^3}{GM}} = \sqrt{\frac{4\pi^2 (300 \times 10^9 \text{ m})^3}{(6.67 \times 10^{-11} \text{m}^3/\text{s}^2 \cdot \text{kg})(1.99 \times 10^{30} \text{kg})}} = 8.96 \times 10^7 \text{ s}.$$

Dividing by (365 d/y) (24 h/d) (60 min/h) (60 s/min), we obtain $T = 2.8$ y.

(b) The kinetic energy of any asteroid or planet in a circular orbit of radius r is given by $K = GMm/2r$, where m is the mass of the asteroid or planet. We note that it is proportional to m and inversely proportional to r. The ratio of the kinetic energy of the asteroid to the kinetic energy of Earth is $K/K_e = (m/m_e)\,(r_e/r)$. We substitute $m = 2.0 \times 10^{-4} m_e$ and $r = 2r_e$ to obtain $K/K_e = 1.0 \times 10^{-4}$.

62. (a) Circular motion requires that the force in Newton's second law provide the necessary centripetal acceleration:

$$\frac{GmM}{r^2} = m\frac{v^2}{r}.$$

Since the left-hand side of this equation is the force given as 80 N, then we can solve for the combination mv^2 by multiplying both sides by $r = 2.0 \times 10^7$ m. Thus, $mv^2 = (2.0 \times 10^7$ m) (80 N) $= 1.6 \times 10^9$ J. Therefore,

$$K = \frac{1}{2}mv^2 = \frac{1}{2}\left(1.6 \times 10^9 \text{ J}\right) = 8.0 \times 10^8 \text{ J}.$$

(b) Since the gravitational force is inversely proportional to the square of the radius, then

$$\frac{F'}{F} = \left(\frac{r}{r'}\right)^2.$$

Thus, $F' = (80 \text{ N})(2/3)^2 = 36$ N.

63. The energy required to raise a satellite of mass *m* to an altitude *h* (at rest) is given by

$$E_1 = \Delta U = GM_E m\left(\frac{1}{R_E} - \frac{1}{R_E + h}\right),$$

and the energy required to put it in circular orbit once it is there is

$$E_2 = \frac{1}{2}mv_{\text{orb}}^2 = \frac{GM_E m}{2(R_E + h)}.$$

Consequently, the energy difference is

$$\Delta E = E_1 - E_2 = GM_E m\left[\frac{1}{R_E} - \frac{3}{2(R_E + h)}\right].$$

(a) Solving the above equation, the height h_0 at which $\Delta E = 0$ is given by

$$\frac{1}{R_E} - \frac{3}{2(R_E + h_0)} = 0 \quad \Rightarrow h_0 = \frac{R_E}{2} = 3.19\times10^6 \text{ m}.$$

(b) For greater height $h > h_0$, $\Delta E > 0$ implying $E_1 > E_2$. Thus, the energy of lifting is greater.

64. (a) From Eq. 13-40, we see that the energy of each satellite is $-GM_Em/2r$. The total energy of the two satellites is twice that result:

$$E = E_A + E_B = -\frac{GM_E m}{r} = -\frac{(6.67\times10^{-11}\ \text{m}^3/\text{kg}\cdot\text{s}^2)(5.98\times10^{24}\text{kg})(125\ \text{kg})}{7.87\times10^6\ \text{m}}$$
$$= -6.33\times10^9\,\text{J}.$$

(b) We note that the speed of the wreckage will be zero (immediately after the collision), so it has no kinetic energy at that moment. Replacing m with $2m$ in the potential energy expression, we therefore find the total energy of the wreckage at that instant is

$$E = -\frac{GM_E(2m)}{2r} = -\frac{(6.67\times10^{-11}\ \text{m}^3/\text{kg}\cdot\text{s}^2)(5.98\times10^{24}\text{kg})2(125\ \text{kg})}{2(7.87\times10^6\ \text{m})} = -6.33\times10^9\,\text{J}.$$

(c) An object with zero speed at that distance from Earth will simply fall towards the Earth, its trajectory being toward the center of the planet.

65. (a) From Kepler's law of periods, we see that T is proportional to $r^{3/2}$.

(b) Eq. 13-38 shows that K is inversely proportional to r.

(c) and (d) From the previous part, knowing that K is proportional to v^2, we find that v is proportional to $1/\sqrt{r}$. Thus, by Eq. 13-31, the angular momentum (which depends on the product rv) is proportional to $r/\sqrt{r} = \sqrt{r}$.

66. (a) The pellets will have the same speed v but opposite direction of motion, so the *relative speed* between the pellets and satellite is $2v$. Replacing v with $2v$ in Eq. 13-38 is equivalent to multiplying it by a factor of 4. Thus,

$$K_{\text{rel}} = 4\left(\frac{GM_E m}{2r}\right) = \frac{2(6.67\times10^{-11}\ \text{m}^3/\text{kg}\cdot\text{s}^2)\left(5.98\times10^{24}\ \text{kg}\right)\left(0.0040\ \text{kg}\right)}{(6370+500)\times10^3\ \text{m}} = 4.6\times10^5\ \text{J}.$$

(b) We set up the ratio of kinetic energies:

$$\frac{K_{\text{rel}}}{K_{\text{bullet}}} = \frac{4.6\times10^5\ \text{J}}{\frac{1}{2}\left(0.0040\,\text{kg}\right)\left(950\,\text{m/s}\right)^2} = 2.6\times10^2.$$

67. (a) The force acting on the satellite has magnitude GMm/r^2, where M is the mass of Earth, m is the mass of the satellite, and r is the radius of the orbit. The force points toward the center of the orbit. Since the acceleration of the satellite is v^2/r, where v is its speed, Newton's second law yields $GMm/r^2 = mv^2/r$ and the speed is given by $v = \sqrt{GM/r}$. The radius of the orbit is the sum of Earth's radius and the altitude of the satellite: $r = (6.37 \times 10^6 + 640 \times 10^3)$ m $= 7.01 \times 10^6$ m. Thus,

$$v = \sqrt{\frac{(6.67 \times 10^{-11}\ \mathrm{m^3/s^2 \cdot kg})\left(5.98 \times 10^{24}\ \mathrm{kg}\right)}{7.01 \times 10^6\ \mathrm{m}}} = 7.54 \times 10^3\ \mathrm{m/s}.$$

(b) The period is

$$T = 2\pi r/v = 2\pi(7.01 \times 10^6\ \mathrm{m})/(7.54 \times 10^3\ \mathrm{m/s}) = 5.84 \times 10^3\ \mathrm{s} \approx 97\ \mathrm{min}.$$

(c) If E_0 is the initial energy then the energy after n orbits is $E = E_0 - nC$, where $C = 1.4 \times 10^5$ J/orbit. For a circular orbit the energy and orbit radius are related by $E = -GMm/2r$, so the radius after n orbits is given by $r = -GMm/2E$.
The initial energy is

$$E_0 = -\frac{(6.67 \times 10^{-11}\ \mathrm{m^3/s^2 \cdot kg})\left(5.98 \times 10^{24}\ \mathrm{kg}\right)(220\ \mathrm{kg})}{2(7.01 \times 10^6\ \mathrm{m})} = -6.26 \times 10^9\ \mathrm{J},$$

the energy after 1500 orbits is

$$E = E_0 - nC = -6.26 \times 10^9\ \mathrm{J} - (1500\ \mathrm{orbit})\left(1.4 \times 10^5\ \mathrm{J/orbit}\right) = -6.47 \times 10^9\ \mathrm{J},$$

and the orbit radius after 1500 orbits is

$$r = -\frac{(6.67 \times 10^{-11}\ \mathrm{m^3/s^2 \cdot kg})\left(5.98 \times 10^{24}\ \mathrm{kg}\right)(220\ \mathrm{kg})}{2(-6.47 \times 10^9\ \mathrm{J})} = 6.78 \times 10^6\ \mathrm{m}.$$

The altitude is $h = r - R = (6.78 \times 10^6\ \mathrm{m} - 6.37 \times 10^6\ \mathrm{m}) = 4.1 \times 10^5$ m. Here R is the radius of Earth. This torque is internal to the satellite-Earth system, so the angular momentum of that system is conserved.

(d) The speed is

$$v = \sqrt{\frac{GM}{r}} = \sqrt{\frac{(6.67 \times 10^{-11}\ \mathrm{m^3/s^2 \cdot kg})\left(5.98 \times 10^{24}\ \mathrm{kg}\right)}{6.78 \times 10^6\ \mathrm{m}}} = 7.67 \times 10^3\ \mathrm{m/s} \approx 7.7\ \mathrm{km/s}.$$

(e) The period is

$$T = \frac{2\pi r}{v} = \frac{2\pi(6.78 \times 10^6\ \mathrm{m})}{7.67 \times 10^3\ \mathrm{m/s}} = 5.6 \times 10^3\ \mathrm{s} \approx 93\ \mathrm{min}.$$

(f) Let F be the magnitude of the average force and s be the distance traveled by the satellite. Then, the work done by the force is $W = -Fs$. This is the change in energy: $-Fs = \Delta E$. Thus, $F = -\Delta E/s$. We evaluate this expression for the first orbit. For a complete orbit $s = 2\pi r = 2\pi(7.01 \times 10^6 \text{ m}) = 4.40 \times 10^7$ m, and $\Delta E = -1.4 \times 10^5$ J. Thus,

$$F = -\frac{\Delta E}{s} = \frac{1.4 \times 10^5 \text{ J}}{4.40 \times 10^7 \text{ m}} = 3.2 \times 10^{-3} \text{ N}.$$

(g) The resistive force exerts a torque on the satellite, so its angular momentum is not conserved.

(h) The satellite-Earth system is essentially isolated, so its momentum is very nearly conserved.

68. The orbital radius is $r = R_E + h = 6370\text{ km} + 400\text{ km} = 6770\text{ km} = 6.77\times10^6\text{ m}$.

(a) Using Kepler's law given in Eq. 13-34, we find the period of the ships to be

$$T_0 = \sqrt{\frac{4\pi^2 r^3}{GM}} = \sqrt{\frac{4\pi^2 (6.77\times10^6\text{ m})^3}{(6.67\times10^{-11}\text{m}^3/\text{s}^2\cdot\text{kg})(5.98\times10^{24}\text{kg})}} = 5.54\times10^3\text{ s} \approx 92.3\text{ min}.$$

(b) The speed of the ships is

$$v_0 = \frac{2\pi r}{T_0} = \frac{2\pi(6.77\times10^6\text{ m})}{5.54\times10^3\text{ s}} = 7.68\times10^3\text{ m/s}^2.$$

(c) The new kinetic energy is

$$K = \frac{1}{2}mv^2 = \frac{1}{2}m(0.99v_0)^2 = \frac{1}{2}(2000\text{ kg})(0.99)^2(7.68\times10^3\text{ m/s})^2 = 5.78\times10^{10}\text{ J}.$$

(d) Immediately after the burst, the potential energy is the same as it was before the burst. Therefore,

$$U = -\frac{GMm}{r} = -\frac{(6.67\times10^{-11}\text{m}^3/\text{s}^2\cdot\text{kg})(5.98\times10^{24}\text{kg})(2000\text{ kg})}{6.77\times10^6\text{ m}} = -1.18\times10^{11}\text{ J}.$$

(e) In the new elliptical orbit, the total energy is

$$E = K + U = 5.78\times10^{10}\text{ J} + (-1.18\times10^{11}\text{ J}) = -6.02\times10^{10}\text{ J}.$$

(f) For elliptical orbit, the total energy can be written as (see Eq. 13-42) $E = -GMm/2a$, where a is the semi-major axis. Thus,

$$a = -\frac{GMm}{2E} = -\frac{(6.67\times10^{-11}\text{m}^3/\text{s}^2\cdot\text{kg})(5.98\times10^{24}\text{kg})(2000\text{ kg})}{2(-6.02\times10^{10}\text{ J})} = 6.63\times10^6\text{ m}.$$

(g) To find the period, we use Eq. 13-34 but replace r with a. The result is

$$T = \sqrt{\frac{4\pi^2 a^3}{GM}} = \sqrt{\frac{4\pi^2 (6.63\times10^6\text{ m})^3}{(6.67\times10^{-11}\text{m}^3/\text{s}^2\cdot\text{kg})(5.98\times10^{24}\text{kg})}} = 5.37\times10^3\text{ s} \approx 89.5\text{ min}.$$

(h) The orbital period T for Picard's elliptical orbit is shorter than Igor's by

$$\Delta T = T_0 - T = 5540\text{ s} - 5370\text{ s} = 170\text{ s}.$$

Thus, Picard will arrive back at point P ahead of Igor by 170 s – 90 s = 80 s.

69. We define the “effective gravity” in his environment as g_{eff} = 220/60 = 3.67 m/s^2. Thus, using equations from Chapter 2 (and selecting downwards as the positive direction), we find the “fall-time” to be

$$\Delta y = v_0 t + \frac{1}{2} g_{eff} t^2 \Rightarrow t = \sqrt{\frac{2(2.1 \text{ m})}{3.67 \text{ m/s}^2}} = 1.1 \text{ s}.$$

70. We estimate the planet to have radius $r = 10$ m. To estimate the mass m of the planet, we require its density equal that of Earth (and use the fact that the volume of a sphere is $4\pi r^3/3$):

$$\frac{m}{4\pi r^3/3} = \frac{M_E}{4\pi R_E^3/3} \quad \Rightarrow \quad m = M_E\left(\frac{r}{R_E}\right)^3$$

which yields (with $M_E \approx 6 \times 10^{24}$ kg and $R_E \approx 6.4 \times 10^6$ m) $m = 2.3 \times 10^7$ kg.

(a) With the above assumptions, the acceleration due to gravity is

$$a_g = \frac{Gm}{r^2} = \frac{\left(6.7\times10^{-11}\ \text{m}^3/\text{s}^2\cdot\text{kg}\right)\left(2.3\times10^7\ \text{kg}\right)}{(10\ \text{m})^2} = 1.5\times10^{-5}\ \text{m/s}^2 \approx 2\times10^{-5}\ \text{m/s}^2 .$$

(b) Eq. 13-28 gives the escape speed:

$$v = \sqrt{\frac{2Gm}{r}} \approx 0.02\ \text{m/s} .$$

71. Using energy conservation (and Eq. 13-21) we have

$$K_1 - \frac{GMm}{r_1} = K_2 - \frac{GMm}{r_2} \quad .$$

Plugging in two pairs of values (for (K_1, r_1) and (K_2, r_2)) from the graph and using the value of G and M (for earth) given in the book, we find

(a) $m \approx 1.0 \times 10^3$ kg.

(b) Similarly, $v = (2K/m)^{1/2} \approx 1.5 \times 10^3$ m/s (at $r = 1.945 \times 10^7$ m).

72. (a) The gravitational acceleration a_g is defined in Eq. 13-11. The problem is concerned with the difference between a_g evaluated at $r = 50R_h$ and a_g evaluated at $r = 50R_h + h$ (where h is the estimate of your height). Assuming h is much smaller than $50R_h$ then we can approximate h as the dr which is present when we consider the differential of Eq. 13-11:

$$|da_g| = \frac{2GM}{r^3}\,dr \approx \frac{2GM}{50^3 R_h^{\,3}}\,h = \frac{2GM}{50^3(2GM/c^2)^3}\,h\,.$$

If we approximate $|da_g| = 10$ m/s^2 and $h \approx 1.5$ m, we can solve this for M. Giving our results in terms of the Sun's mass means dividing our result for M by 2×10^{30} kg. Thus, admitting some tolerance into our estimate of h we find the "critical" black hole mass should in the range of 105 to 125 solar masses.

(b) Interestingly, this turns out to be lower limit (which will surprise many students) since the above expression shows $|da_g|$ is inversely proportional to M. It should perhaps be emphasized that a distance of $50R_h$ from a small black hole is much smaller than a distance of $50R_h$ from a large black hole.

73. The magnitudes of the individual forces (acting on m_C, exerted by m_A and m_B respectively) are

$$F_{AC} = \frac{Gm_A\, m_C}{r_{AC}^2} = 2.7 \times 10^{-8}\ \text{N} \quad \text{and} \quad F_{BC} = \frac{Gm_B\, m_C}{r_{BC}^2} = 3.6 \times 10^{-8}\ \text{N}$$

where r_{AC} = 0.20 m and r_{BC} = 0.15 m. With r_{AB} = 0.25 m, the angle $\vec{F}_A$ makes with the x axis can be obtained as

$$\theta_A = \pi + \cos^{-1}\left(\frac{r_{AC}^2 + r_{AB}^2 - r_{BC}^2}{2r_{AC}r_{AB}}\right) = \pi + \cos^{-1}(0.80) = 217°.$$

Similarly, the angle $\vec{F}_B$ makes with the x axis can be obtained as

$$\theta_B = -\cos^{-1}\left(\frac{r_{AB}^2 + r_{BC}^2 - r_{AC}^2}{2r_{AB}r_{BC}}\right) = -\cos^{-1}(0.60) = -53°.$$

The net force acting on m_C then becomes

$$\begin{aligned}\vec{F}_C &= F_{AC}(\cos\theta_A\,\hat{\text{i}} + \sin\theta_A\,\hat{\text{j}}) + F_{BC}(\cos\theta_B\,\hat{\text{i}} + \sin\theta_B\,\hat{\text{j}})\\ &= (F_{AC}\cos\theta_A + F_{BC}\cos\theta_B)\hat{\text{i}} + (F_{AC}\sin\theta_A + F_{BC}\sin\theta_B)\hat{\text{j}}\\ &= (-4.4\times 10^{-8}\ \text{N})\hat{\text{j}}\end{aligned}$$

74. The key point here is that angular momentum is conserved:

$$I_p\omega_p = I_a\omega_a$$

which leads to $\omega_p = (r_a / r_p)^2 \omega_a$, but $r_p = 2a - r_a$ where a is determined by Eq. 13-34 (particularly, see the paragraph after that equation in the textbook). Therefore,

$$\omega_p = \frac{r_a^{\ 2}\,\omega_a}{(2(GMT^2/4\pi^2)^{1/3} - r_a)^2} = 9.24 \times 10^{-5} \text{ rad/s} .$$

75. (a) Using Kepler's law of periods, we obtain

$$T = \sqrt{\left(\frac{4\pi^2}{GM}\right) r^3} = 2.15 \times 10^4 \text{ s} .$$

(b) The speed is constant (before she fires the thrusters), so $v_o = 2\pi r/T = 1.23 \times 10^4$ m/s.

(c) A two percent reduction in the previous value gives $v = 0.98v_o = 1.20 \times 10^4$ m/s.

(d) The kinetic energy is $K = \frac{1}{2}mv^2 = 2.17 \times 10^{11}$ J.

(e) The potential energy is $U = -GmM/r = -4.53 \times 10^{11}$ J.

(f) Adding these two results gives $E = K + U = -2.35 \times 10^{11}$ J.

(g) Using Eq. 13-42, we find the semi-major axis to be

$$a = \frac{-GMm}{2E} = 4.04 \times 10^7 \text{ m} .$$

(h) Using Kepler's law of periods for elliptical orbits (using a instead of r) we find the new period is

$$T' = \sqrt{\left(\frac{4\pi^2}{GM}\right) a^3} = 2.03 \times 10^4 \text{ s} .$$

This is smaller than our result for part (a) by $T - T' = 1.22 \times 10^3$ s.

(i) Elliptical orbit has a smaller period.

76. (a) With $M = 2.0 \times 10^{30}$ kg and $r = 10000$ m, we find

$$a_g = \frac{GM}{r^2} = 1.3 \times 10^{12} \text{ m/s}^2 .$$

(b) Although a close answer may be gotten by using the constant acceleration equations of Chapter 2, we show the more general approach (using energy conservation):

$$K_o + U_o = K + U$$

where $K_o = 0$, $K = \frac{1}{2}mv^2$ and U given by Eq. 13-21. Thus, with $r_o = 10001$ m, we find

$$v = \sqrt{2GM\left(\frac{1}{r} - \frac{1}{r_o}\right)} = 1.6 \times 10^6 \text{ m/s} .$$

77. We note that r_A (the distance from the origin to sphere A, which is the same as the separation between A and B) is 0.5, $r_C = 0.8$, and $r_D = 0.4$ (with SI units understood). The force $\vec{F}_k$ that the k^{th} sphere exerts on m_B has magnitude $Gm_k m_B / r_k^2$ and is directed from the origin towards m_k so that it is conveniently written as

$$\vec{F}_k = \frac{Gm_k m_B}{r_k^2}\left(\frac{x_k}{r_k}\hat{\text{i}} + \frac{y_k}{r_k}\hat{\text{j}}\right) = \frac{Gm_k m_B}{r_k^3}\left(x_k\hat{\text{i}} + y_k\hat{\text{j}}\right).$$

Consequently, the vector addition (where k equals A,B and D) to obtain the net force on m_B becomes

$$\vec{F}_{\text{net}} = \sum_k \vec{F}_k = Gm_B\left(\left(\sum_k \frac{m_k x_k}{r_k^3}\right)\hat{\text{i}} + \left(\sum_k \frac{m_k y_k}{r_k^3}\right)\hat{\text{j}}\right) = (3.7\times10^{-5}\,\text{N})\hat{\text{j}}.$$

78. (a) We note that r_C (the distance from the origin to sphere C, which is the same as the separation between C and B) is 0.8, $r_D = 0.4$, and the separation between spheres C and D is $r_{CD} = 1.2$ (with SI units understood). The total potential energy is therefore

$$-\frac{GM_BM_C}{r_C^2}-\frac{GM_BM_D}{r_D^2}-\frac{GM_CM_D}{r_{CD}^2}=-1.3\times10^{-4}\ \text{J}$$

using the mass-values given in the previous problem.

(b) Since any gravitational potential energy term (of the sort considered in this chapter) is necessarily negative ($-GmM/r^2$ where all variables are positive) then having another mass to include in the computation can only lower the result (that is, make the result more negative).

(c) The observation in the previous part implies that the work I do in removing sphere A (to obtain the case considered in part (a)) must lead to an increase in the system energy; thus, I do positive work.

(d) To put sphere A back in, I do negative work, since I am causing the system energy to become more negative.

79. We use $F = Gm_s m_m/r^2$, where m_s is the mass of the satellite, m_m is the mass of the meteor, and r is the distance between their centers. The distance between centers is $r = R + d = 15\text{ m} + 3\text{ m} = 18\text{ m}$. Here R is the radius of the satellite and d is the distance from its surface to the center of the meteor. Thus,

$$F = \frac{\left(6.67\times10^{-11}\,\text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(20\text{kg}\right)\left(7.0\text{kg}\right)}{\left(18\text{m}\right)^2} = 2.9\times10^{-11}\,\text{N}.$$

80. (a) Since the volume of a sphere is $4\pi R^3/3$, the density is

$$\rho = \frac{M_{\text{total}}}{\frac{4}{3}\pi R^3} = \frac{3M_{\text{total}}}{4\pi R^3}.$$

When we test for gravitational acceleration (caused by the sphere, or by parts of it) at radius r (measured from the center of the sphere), the mass M which is at radius less than r is what contributes to the reading (GM/r^2). Since $M = \rho(4\pi r^3/3)$ for $r \le R$ then we can write this result as

$$\frac{G\left(\dfrac{3M_{\text{total}}}{4\pi R^3}\right)\left(\dfrac{4\pi r^3}{3}\right)}{r^2} = \frac{GM_{\text{total}}r}{R^3}$$

when we are considering points on or inside the sphere. Thus, the value a_g referred to in the problem is the case where $r = R$:

$$a_g = \frac{GM_{\text{total}}}{R^2},$$

and we solve for the case where the acceleration equals $a_g/3$:

$$\frac{GM_{\text{total}}}{3R^2} = \frac{GM_{\text{total}}r}{R^3} \quad \Rightarrow \quad r = \frac{R}{3}.$$

(b) Now we treat the case of an external test point. For points with $r > R$ the acceleration is GM_{total}/r^2, so the requirement that it equal $a_g/3$ leads to

$$\frac{GM_{\text{total}}}{3R^2} = \frac{GM_{\text{total}}}{r^2} \quad \Rightarrow \quad r = \sqrt{3}R.$$

81. Energy conservation for this situation may be expressed as follows:

$$K_1 + U_1 = K_2 + U_2 \quad \Rightarrow \quad \frac{1}{2}mv_1^2 - \frac{GmM}{r_1} = \frac{1}{2}mv_2^2 - \frac{GmM}{r_2}$$

where $M = 5.98 \times 10^{24}$ kg, $r_1 = R = 6.37 \times 10^6$ m and $v_1 = 10000$ m/s. Setting $v_2 = 0$ to find the maximum of its trajectory, we solve the above equation (noting that m cancels in the process) and obtain $r_2 = 3.2 \times 10^7$ m. This implies that its *altitude* is $r_2 - R = 2.5 \times 10^7$ m.

82. (a) Because it is moving in a circular orbit, F/m must equal the centripetal acceleration:

$$\frac{80\ \text{N}}{50\ \text{kg}} = \frac{v^2}{r}.$$

But $v = 2\pi r/T$, where $T = 21600$ s, so we are led to

$$1.6\,\text{m/s}^2 = \frac{4\pi^2}{T^2} r$$

which yields $r = 1.9 \times 10^7$ m.

(b) From the above calculation, we infer $v^2 = (1.6\ \text{m/s}^2)r$ which leads to $v^2 = 3.0 \times 10^7$ m^2/s^2. Thus, $K = \tfrac{1}{2}mv^2 = 7.6 \times 10^8$ J.

(c) As discussed in § 13-4, F/m also tells us the gravitational acceleration:

$$a_g = 1.6\ \text{m/s}^2 = \frac{GM}{r^2}.$$

We therefore find $M = 8.6 \times 10^{24}$ kg.

83. (a) We write the centripetal acceleration (which is the same for each, since they have identical mass) as $r\omega^2$ where ω is the unknown angular speed. Thus,

$$\frac{G(M)(M)}{(2r)^2} = \frac{GM^2}{4r^2} = Mr\omega^2$$

which gives $\omega = \frac{1}{2}\sqrt{MG/r^3} = 2.2\times10^{-7}$ rad/s.

(b) To barely escape means to have total energy equal to zero (see discussion prior to Eq. 13-28). If m is the mass of the meteoroid, then

$$\frac{1}{2}mv^2 - \frac{GmM}{r} - \frac{GmM}{r} = 0 \;\Rightarrow\; v = \sqrt{\frac{4GM}{r}} = 8.9\times10^4 \text{ m/s} .$$

84. See Appendix C. We note that, since $v = 2\pi r/T$, the centripetal acceleration may be written as $a = 4\pi^2 r/T^2$. To express the result in terms of g, we divide by 9.8 m/s^2.

(a) The acceleration associated with Earth's spin (T = 24 h = 86400 s) is

$$a = g\frac{4\pi^2\,(6.37\times10^6\text{ m})}{(86400\,\text{s})^2\,(9.8\,\text{m/s}^2)} = 3.4\times10^{-3}\,g\ .$$

(b) The acceleration associated with Earth's motion around the Sun (T = 1 y = 3.156×10^7 s) is

$$a = g\frac{4\pi^2\,(1.5\times10^{11}\text{ m})}{(3.156\times10^7\text{ s})^2\,(9.8\,\text{m/s}^2)} = 6.1\times10^{-4}\,g\ .$$

(c) The acceleration associated with the Solar System's motion around the galactic center ($T = 2.5 \times 10^8$ y $= 7.9 \times 10^{15}$ s) is

$$a = g\frac{4\pi^2\,(2.2\times10^{20}\text{ m})}{(7.9\times10^{15}\text{ s})^2\,(9.8\,\text{m/s}^2)} = 1.4\times10^{-11}\,g\ .$$

85. We use m_1 for the 20 kg of the sphere at $(x_1, y_1) = (0.5, 1.0)$ (SI units understood), m_2 for the 40 kg of the sphere at $(x_2, y_2) = (-1.0, -1.0)$, and m_3 for the 60 kg of the sphere at $(x_3, y_3) = (0, -0.5)$. The mass of the 20 kg object at the origin is simply denoted m. We note that $r_1 = \sqrt{1.25}$, $r_2 = \sqrt{2}$, and $r_3 = 0.5$ (again, with SI units understood). The force $\vec{F}_n$ that the n^{th} sphere exerts on m has magnitude $Gm_n m / r_n^2$ and is directed from the origin towards m_n, so that it is conveniently written as

$$\vec{F}_n = \frac{Gm_n m}{r_n^2}\left(\frac{x_n}{r_n}\hat{\mathrm{i}} + \frac{y_n}{r_n}\hat{\mathrm{j}}\right) = \frac{Gm_n m}{r_n^3}\left(x_n\hat{\mathrm{i}} + y_n\hat{\mathrm{j}}\right).$$

Consequently, the vector addition to obtain the net force on m becomes

$$\vec{F}_{\text{net}} = \sum_{n=1}^{3}\vec{F}_n = Gm\left(\left(\sum_{n=1}^{3}\frac{m_n x_n}{r_n^3}\right)\hat{\mathrm{i}} + \left(\sum_{n=1}^{3}\frac{m_n y_n}{r_n^3}\right)\hat{\mathrm{j}}\right) = -9.3\times10^{-9}\,\hat{\mathrm{i}} - 3.2\times10^{-7}\,\hat{\mathrm{j}}$$

in SI units. Therefore, we find the net force magnitude is $\left|\vec{F}_{\text{net}}\right| = 3.2\times10^{-7}\,\mathrm{N}$.

86. We apply the work-energy theorem to the object in question. It starts from a point at the surface of the Earth with zero initial speed and arrives at the center of the Earth with final speed v_f. The corresponding increase in its kinetic energy, $\frac{1}{2}mv_f^2$, is equal to the work done on it by Earth's gravity: $\int F\,dr = \int(-Kr)dr$ (using the notation of that Sample Problem referred to in the problem statement). Thus,

$$\frac{1}{2}mv_f^2 = \int_R^0 F\,dr = \int_R^0 (-Kr)\,dr = \frac{1}{2}KR^2$$

where R is the radius of Earth. Solving for the final speed, we obtain $v_f = R\sqrt{K/m}$. We note that the acceleration of gravity $a_g = g = 9.8\text{ m/s}^2$ on the surface of Earth is given by

$$a_g = GM/R^2 = G(4\pi R^3/3)\rho/R^2,$$

where ρ is Earth's average density. This permits us to write $K/m = 4\pi G\rho/3 = g/R$. Consequently,

$$v_f = R\sqrt{\frac{K}{m}} = R\sqrt{\frac{g}{R}} = \sqrt{gR} = \sqrt{(9.8\text{ m/s}^2)(6.37\times10^6\text{ m})} = 7.9\times10^3\text{ m/s}\,.$$

87. (a) The total energy is conserved, so there is no difference between its values at aphelion and perihelion.

(b) Since the change is small, we use differentials:

$$dU = \left(\frac{GM_E M_S}{r^2}\right) dr \approx \left(\frac{\left(6.67\times10^{-11}\right)\left(1.99\times10^{30}\right)\left(5.98\times10^{24}\right)}{\left(1.5\times10^{11}\right)^2}\right)\left(5\times10^{9}\right)$$

which yields $\Delta U \approx 1.8 \times 10^{32}$ J. A more direct subtraction of the values of the potential energies leads to the same result.

(c) From the previous two parts, we see that the variation in the kinetic energy ΔK must also equal 1.8×10^{32} J.

(d) With $\Delta K \approx dK = mv\, dv$, where $v \approx 2\pi R/T$, we have

$$1.8\times10^{32} \approx \left(5.98\times10^{24}\right)\left(\frac{2\pi\left(1.5\times10^{11}\right)}{3.156\times10^{7}}\right)\Delta v$$

which yields a difference of $\Delta v \approx 0.99$ km/s in Earth's speed (relative to the Sun) between aphelion and perihelion.

88. Let the distance from Earth to the spaceship be r. $R_{em} = 3.82 \times 10^8$ m is the distance from Earth to the moon. Thus,

$$F_m = \frac{GM_m m}{\left(R_{em} - r\right)^2} = F_E = \frac{GM_e m}{r^2},$$

where m is the mass of the spaceship. Solving for r, we obtain

$$r = \frac{R_{em}}{\sqrt{M_m / M_e} + 1} = \frac{3.82\times10^8\,\text{m}}{\sqrt{(7.36\times10^{22}\,\text{kg})/(5.98\times10^{24}\,\text{kg})} + 1} = 3.44\times10^8\,\text{m}.$$

89. We integrate Eq. 13-1 with respect to r from $3R_E$ to $4R_E$ and obtain the work equal to $-GM_Em(1/(4R_E) - 1/(3R_E)) = GM_Em/12R_E$.

90. If the angular velocity were any greater, loose objects on the surface would not go around with the planet but would travel out into space.

(a) The magnitude of the gravitational force exerted by the planet on an object of mass m at its surface is given by $F = GmM / R^2$, where M is the mass of the planet and R is its radius. According to Newton's second law this must equal mv^2 / R, where v is the speed of the object. Thus,

$$\frac{GM}{R^2} = \frac{v^2}{R}.$$

Replacing M with $(4\pi/3)\,\rho R^3$ (where ρ is the density of the planet) and v with $2\pi R/T$ (where T is the period of revolution), we find

$$\frac{4\pi}{3} G\rho R = \frac{4\pi^2 R}{T^2}.$$

We solve for T and obtain

$$T = \sqrt{\frac{3\pi}{G\rho}}.$$

(b) The density is 3.0×10^3 kg/m^3. We evaluate the equation for T:

$$T = \sqrt{\frac{3\pi}{\left(6.67\times10^{-11}\,\text{m}^3/\text{s}^2\cdot\text{kg}\right)\left(3.0\times10^3\,\text{kg/m}^3\right)}} = 6.86\times10^3\,\text{s} = 1.9\,\text{h}.$$

91. (a) It is possible to use $v^2 = v_0^2 + 2a\,\Delta\,y$ as we did for free-fall problems in Chapter 2 because the acceleration can be considered approximately constant over this interval. However, our approach will not assume constant acceleration; we use energy conservation:

$$\frac{1}{2}mv_0^2 - \frac{GMm}{r_0} = \frac{1}{2}mv^2 - \frac{GMm}{r} \Rightarrow v = \sqrt{\frac{2GM(r_0 - r)}{r_0\,r}}$$

which yields $v = 1.4 \times 10^6$ m/s.

(b) We estimate the height of the apple to be h = 7 cm = 0.07 m. We may find the answer by evaluating Eq. 13-11 at the surface (radius r in part (a)) and at radius $r + h$, being careful not to round off, and then taking the difference of the two values, or we may take the differential of that equation — setting dr equal to h. We illustrate the latter procedure:

$$|\,da_g\,| = \left|-2\frac{GM}{r^3}dr\right| \approx 2\frac{GM}{r^3}h = 3 \times 10^6 \text{ m/s}^2 .$$

92. (a) The gravitational force exerted on the baby (denoted with subscript b) by the obstetrician (denoted with subscript o) is given by

$$F_{bo} = \sqrt{\frac{Gm_o m_b}{r_{bo}^2}} = \sqrt{\frac{\left(6.67\times10^{-11}\,\text{N}\cdot\text{m}^2/\text{kg}^2\right)(70\text{kg})(3\text{kg})}{(1\text{m})^2}} = 1\times10^{-8}\,\text{N}.$$

(b) The maximum (minimum) forces exerted by Jupiter on the baby occur when it is separated from the Earth by the shortest (longest) distance $r_{\min}$ ($r_{\max}$), respectively. Thus

$$F_{bJ}^{\max} = \sqrt{\frac{Gm_J m_b}{r_{\min}^2}} = \sqrt{\frac{\left(6.67\times10^{-11}\,\text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(2\times10^{27}\,\text{kg}\right)(3\text{kg})}{\left(6\times10^{11}\,\text{m}\right)^2}} = 1\times10^{-6}\,\text{N}.$$

(c) And we obtain

$$F_{bJ}^{\min} = \sqrt{\frac{Gm_J m_b}{r_{\max}^2}} = \sqrt{\frac{\left(6.67\times10^{-11}\,\text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(2\times10^{27}\,\text{kg}\right)(3\text{kg})}{\left(9\times10^{11}\,\text{m}\right)^2}} = 5\times10^{-7}\,\text{N}.$$

(d) No. The gravitational force exerted by Jupiter on the baby is greater than that by the obstetrician by a factor of up to 1×10^{-6} N/1×10^{-8} N = 100.

93. The magnitude of the net gravitational force on one of the smaller stars (of mass *m*) is

$$\frac{GMm}{r^2} + \frac{Gmm}{(2r)^2} = \frac{Gm}{r^2}\left(M + \frac{m}{4}\right).$$

This supplies the centripetal force needed for the motion of the star:

$$\frac{Gm}{r^2}\left(M + \frac{m}{4}\right) = m\frac{v^2}{r} \quad \text{where} \ \ v = \frac{2\mathrm{p}r}{T}.$$

Plugging in for speed *v*, we arrive at an equation for period *T*:

$$T = \frac{2\pi r^{3/2}}{\sqrt{G(M + m/4)}}.$$

94. (a) We note that *height* = $R - R_{\text{Earth}}$ where $R_{\text{Earth}} = 6.37 \times 10^6$ m. With $M = 5.98 \times 10^{24}$ kg, $R_0 = 6.57 \times 10^6$ m and $R = 7.37 \times 10^6$ m, we have

$$K_i + U_i = K + U \Rightarrow \frac{1}{2}m\,(3.70 \times 10^3)^2 - \frac{GmM}{R_0} = K - \frac{GmM}{R},$$

which yields $K = 3.83 \times 10^7$ J.

(b) Again, we use energy conservation.

$$K_i + U_i = K_f + U_f \Rightarrow \frac{1}{2}m\,(3.70 \times 10^3)^2 - \frac{GmM}{R_0} = 0 - \frac{GmM}{R_f}$$

Therefore, we find $R_f = 7.40 \times 10^6$ m. This corresponds to a distance of 1034.9 km $\approx 1.03 \times 10^3$ km above the Earth's surface.

95. Energy conservation for this situation may be expressed as follows:

$$K_1 + U_1 = K_2 + U_2 \;\Rightarrow\; \frac{1}{2}mv_1^2 - \frac{GmM}{r_1} = \frac{1}{2}mv_2^2 - \frac{GmM}{r_2}$$

where $M = 7.0 \times 10^{24}$ kg, $r_2 = R = 1.6 \times 10^6$ m and $r_1 = \infty$ (which means that $U_1 = 0$). We are told to assume the meteor starts at rest, so $v_1 = 0$. Thus, $K_1 + U_1 = 0$ and the above equation is rewritten as

$$\frac{1}{2}mv_2^2 - \frac{GmM}{r_2} \quad\Rightarrow\quad v_2 = \sqrt{\frac{2GM}{R}} = 2.4 \times 10^4 \text{ m/s}.$$

96. The initial distance from each fixed sphere to the ball is $r_0 = \infty$, which implies the initial gravitational potential energy is zero. The distance from each fixed sphere to the ball when it is at $x = 0.30$ m is $r = 0.50$ m, by the Pythagorean theorem.

(a) With $M = 20$ kg and $m = 10$ kg, energy conservation leads to

$$K_i + U_i \; = \; K + U \;\; \Rightarrow \;\; 0 + 0 = K - 2\frac{GmM}{r}$$

which yields $K = 2GmM/r = 5.3 \times 10^{-8}$ J.

(b) Since the y-component of each force will cancel, the net force points in the $-x$ direction, with a magnitude $2F_x = 2\,(GmM/r^2)\cos\theta$, where $\theta = \tan^{-1}(4/3) = 53°$. Thus, the result is $\vec{F}_{\text{net}} = (-6.4 \times 10^{-8}\text{ N})\hat{\text{i}}$.

97. The kinetic energy in its circular orbit is $\frac{1}{2}mv^2$ where $v = 2\pi r/T$. Using the values stated in the problem and using Eq. 13-41, we directly find $E = -1.87 \times 10^9$ J.

98. (a) From Ch. 2, we have $v^2 = v_0^2 + 2a\Delta x$, where a may be interpreted as an average acceleration in cases where the acceleration is not uniform. With $v_0 = 0$, $v = 11000$ m/s and $\Delta x = 220$ m, we find $a = 2.75 \times 10^5$ m/s^2. Therefore,

$$a = \left(\frac{2.75 \times 10^5 \text{ m/s}^2}{9.8 \text{ m/s}^2}\right) g = 2.8 \times 10^4\, g\,.$$

(b) The acceleration is certainly deadly enough to kill the passengers.

(c) Again using $v^2 = v_0^2 + 2a\Delta x$, we find

$$a = \frac{(7000 \text{ m/s})^2}{2(3500 \text{ m})} = 7000 \text{ m/s}^2 = 714g\,.$$

(d) Energy conservation gives the craft's speed v (in the absence of friction and other dissipative effects) at altitude $h = 700$ km after being launched from $R = 6.37 \times 10^6$ m (the surface of Earth) with speed $v_0 = 7000$ m/s. That altitude corresponds to a distance from Earth's center of $r = R + h = 7.07 \times 10^6$ m.

$$\frac{1}{2}mv_0^2 - \frac{GMm}{R} = \frac{1}{2}mv^2 - \frac{GMm}{r}\,.$$

With $M = 5.98 \times 10^{24}$ kg (the mass of Earth) we find $v = 6.05 \times 10^3$ m/s. But to orbit at that radius requires (by Eq. 13-37)

$$v' = \sqrt{GM/r} = 7.51 \times 10^3 \text{ m/s}.$$

The difference between these is $v' - v = 1.46 \times 10^3$ m/s $\approx 1.5 \times 10^3$ m/s, which presumably is accounted for by the action of the rocket engine.

99. (a) All points on the ring are the same distance ($r = \sqrt{x^2 + R^2}$) from the particle, so the gravitational potential energy is simply $U = -GMm/\sqrt{x^2 + R^2}$, from Eq. 13-21. The corresponding force (by symmetry) is expected to be along the x axis, so we take a (negative) derivative of U (with respect to x) to obtain it (see Eq. 8-20). The result for the magnitude of the force is $GMmx(x^2 + R^2)^{-3/2}$.

(b) Using our expression for U, then the magnitude of the loss in potential energy as the particle falls to the center is $GMm(1/R - 1/\sqrt{x^2 + R^2}\ )$. This must "turn into" kinetic energy ($\frac{1}{2}mv^2$), so we solve for the speed and obtain

$$v = [2GM(R^{-1} - (R^2 + x^2)^{-1/2})]^{1/2} .$$

100. Consider that we are examining the forces on the mass in the lower left-hand corner of the square. Note that the mass in the upper right-hand corner is $20\sqrt{2} = 28$ cm $= 0.28$ m away. Now, the *nearest* masses each pull with a force of $GmM / r^2 = 3.8 \times 10^{-9}$ N, one upward and the other rightward. The net force caused by these two forces is $(3.8 \times 10^{-9}, 3.8 \times 10^{-9}) \rightarrow (5.3 \times 10^{-9} \angle 45°)$, where the rectangular components are shown first -- and then the polar components (magnitude-angle notation). Now, the mass in the upper right-hand corner also pulls at 45°, so its force-magnitude (1.9×10^{-9}) will simply add to the magnitude just calculated. Thus, the final result is 7.2×10^{-9} N.

101. (a) Their initial potential energy is $-Gm^2/R_i$ and they started from rest, so energy conservation leads to

$$-\frac{Gm^2}{R_i} = K_{\text{total}} - \frac{Gm^2}{0.5R_i} \Rightarrow K_{\text{total}} = \frac{Gm^2}{R_i} .$$

(b) They have equal mass, and this is being viewed in the center-of-mass frame, so their speeds are identical and their kinetic energies are the same. Thus,

$$K = \frac{1}{2} K_{\text{total}} = \frac{Gm^2}{2R_i} .$$

(c) With $K = ½\, mv^2$, we solve the above equation and find $v = \sqrt{Gm/R_i}$.

(d) Their relative speed is $2v = 2\sqrt{Gm/R_i}$. This is the (instantaneous) rate at which the gap between them is closing.

(e) The premise of this part is that we assume we are not moving (that is, that body A acquires no kinetic energy in the process). Thus, $K_{\text{total}} = K_B$ and the logic of part (a) leads to $K_B = Gm^2/R_i$.

(f) And $\frac{1}{2} mv_B^2 = K_B$ yields $v_B = \sqrt{2Gm/R_i}$.

(g) The answer to part (f) is incorrect, due to having ignored the accelerated motion of "our" frame (that of body A). Our computations were therefore carried out in a noninertial frame of reference, for which the energy equations of Chapter 8 are not directly applicable.

102. Gravitational acceleration is defined in Eq. 13-11 (which we are treating as a positive quantity). The problem, then, is asking for the magnitude difference of $a_{g\text{ net}}$ when the contributions from the Moon and the Sun are in the same direction ($a_{g\text{ net}} = a_{g\text{Sun}} + a_{g\text{Moon}}$) as opposed to when they are in opposite directions ($a_{g\text{ net}} = a_{g\text{Sun}} - a_{g\text{Moon}}$). The difference (in absolute value) is clearly $2a_{g\text{Moon}}$. In specifically wanting the *percentage* change, the problem is requesting us to divide this difference by the average of the two $a_{g\text{ net}}$ values being considered (that average is easily seen to be equal to $a_{g\text{Sun}}$), and finally multiply by 100% in order to quote the result in the right format. Thus,

$$\frac{2a_{g\text{Moon}}}{a_{g\text{Sun}}} = 2\left(\frac{M_{\text{Moon}}}{M_{\text{Sun}}}\right)\left(\frac{r_{\text{Sun to Eearth}}}{r_{\text{Moon to Earth}}}\right)^2 = 2\left(\frac{7.36 \times 10^{22}}{1.99 \times 10^{30}}\right)\left(\frac{1.50 \times 10^{11}}{3.82 \times 10^{8}}\right)^2 = 0.011 = 1.1\%.$$

103. (a) Kepler's law of periods is

$$T^2 = \left(\frac{4\pi^2}{GM}\right) r^3 \ .$$

With $M = 6.0 \times 10^{30}$ kg and $T = 300(86400) = 2.6 \times 10^7$ s, we obtain $r = 1.9 \times 10^{11}$ m.

(b) That its orbit is circular suggests that its speed is constant, so

$$v = \frac{2\pi r}{T} = 4.6 \times 10^4 \text{ m/s} \ .$$

104. Using Eq. 13-21, the potential energy of the dust particle is

$$U = -GmM_E/R - GmM_m/r = -Gm(M_E/R + M_m/r) .$$