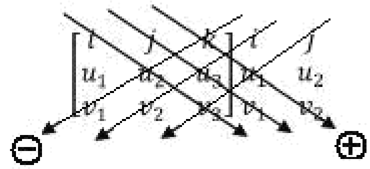
COMPARISON OF PROPERTIES OF DOT PRODUCT AND CROSS PRODUCT

DOT PRODUCT	**CROSS PRODUCT**
$\mathbf{u}\bullet\mathbf{v} = u_1v_1+u_2v_2+u_3v_3$	$\mathbf{u}\times\mathbf{v} = \langle\, u_2v_3-u_3v_2\ ,\ u_3v_1-u_1v_3\ ,\ u_1v_2-u_2v_1 \,\rangle$ $= (u_2v_3-u_3v_2)\mathbf{i} + (u_3v_1-u_1v_3)\mathbf{j} + (u_1v_2-u_2v_1)\mathbf{k}$ $\mathbf{u}\times\mathbf{v}$ can be calculated using diagonal method  $\mathbf{u}\times\mathbf{v}$ can also be calculated by determinant method as shown in the textbook.
vector • vector = scalar (number)	vector × vector = vector
$\mathbf{u}\bullet\mathbf{v}= \lVert\mathbf{u}\rVert\,\lVert\mathbf{v}\rVert \cos\theta$	$\lVert\mathbf{u}\times\mathbf{v}\rVert = \lVert\mathbf{u}\rVert\,\lVert\mathbf{v}\rVert \sin\theta$
$\mathbf{u}\bullet\mathbf{v}$ does not have a "direction" because it is a scalar, not a vector	Direction of $\mathbf{u}\times\mathbf{v}$ is determined by right hand rule: Curl fingers of right hand from **u** to **v;** thumb points in direction of $\mathbf{u}\times\mathbf{v}$
PROPERTIES OF DOT PRODUCT	**PROPERTIES OF CROSS PRODUCT**
$\mathbf{u}\bullet\mathbf{v}= \mathbf{v}\bullet\mathbf{u}$ Commutative	$\mathbf{u}\times\mathbf{v} = -\,\mathbf{v}\times\mathbf{u}$ Anticommutative
$\mathbf{u}\bullet(\mathbf{v}+\mathbf{w})= \mathbf{u}\bullet\mathbf{v} + \mathbf{u}\bullet\mathbf{w}$ Distributive over addition	$\mathbf{u}\times(\mathbf{v}+\mathbf{w})= \mathbf{u}\times\mathbf{v} + \mathbf{u}\times\mathbf{w}$ Distributive over addition
$c\,(\mathbf{u}\bullet\mathbf{v})= (c\mathbf{u})\bullet\mathbf{v} = \mathbf{u}\bullet(c\mathbf{v})$	$c(\mathbf{u}\times\mathbf{v})= (c\mathbf{u})\times\mathbf{v} = \mathbf{u}\times(c\mathbf{v})$
$\mathbf{v}\bullet\mathbf{v} = \lVert\mathbf{v}\rVert^2$	$\mathbf{i}\times\mathbf{j}=\mathbf{k}$ $\quad\mathbf{j}\times\mathbf{k}=\mathbf{i}$ $\quad\mathbf{k}\times\mathbf{i}=\mathbf{j}$
$\mathbf{v}\bullet\mathbf{v} = 0$ if and only if $\mathbf{v} = \mathbf{0}$	$\mathbf{v}\times\mathbf{v} = \mathbf{0}$ (because **v** is parallel to itself)
If **u** and **v** are non-zero vectors then: $\mathbf{u}\bullet\mathbf{v}= 0$ if and only if **u** and **v** are orthogonal $\mathbf{u}\bullet\mathbf{v}>0$ if and only if the angle between **u** and **v** is acute ($0^\circ < \theta < 90^\circ$) $\mathbf{u}\bullet\mathbf{v}<0$ if and only if the angle between **u** and **v** is obtuse ($90^\circ < \theta < 180^\circ$)	If **u** and **v** are non-zero vectors then: $\mathbf{u}\times\mathbf{v}$ is orthogonal to both **u** and **v** $\mathbf{u}\times\mathbf{v} = \mathbf{0}$ if and only if **u** and **v** are parallel
APPLICATIONS OF DOT PRODUCT	**APPLICATIONS OF CROSS PRODUCT**
$\cos\theta=\dfrac{\mathbf{u}\bullet\mathbf{v}}{\lVert\mathbf{u}\rVert\lVert\mathbf{v}\rVert}$ allows us to find the angle between **u** and **v**	Use $\mathbf{u}\times\mathbf{v}$ when you need to find a vector that is orthogonal to both **u** and **v**
$\mathbf{proj_v u} = \left(\dfrac{\mathbf{u}\bullet\mathbf{v}}{\lVert\mathbf{v}\rVert}\right)\dfrac{\mathbf{v}}{\lVert\mathbf{v}\rVert} = \left(\dfrac{\mathbf{u}\bullet\mathbf{v}}{\lVert\mathbf{v}\rVert^2}\right)\mathbf{v} = \left(\dfrac{\mathbf{u}\bullet\mathbf{v}}{\mathbf{v}\bullet\mathbf{v}}\right)\mathbf{v}$ $\mathbf{orth_v u} = \mathbf{u} - \mathbf{proj_v u}$	area of parallelogram defined by vectors **u** and **v** $= \lVert\mathbf{u}\times\mathbf{v}\rVert$ area of triangle defined by vectors **u** and **v** $= (1/2)\lVert\mathbf{u}\times\mathbf{v}\rVert$
Work = $\mathbf{F}\bullet\mathbf{D}$ U.S.: units of work are foot-pounds Metric: units of work are joules	Cross product is useful when finding the equation of a plane that contains two given vectors. (This will be covered in section 11.4)