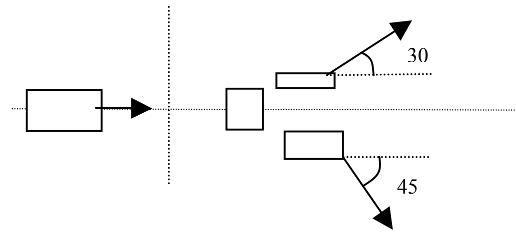
Center of Mass and Momentum Essentials

1. A 10 kg block is sliding along a frictionless surface at 4 m/s when it breaks into two pieces. If a 4 kg piece travels in the original direction at 6 m/s, what is the final velocity of the other piece?

2. A 0.5 kg stone is dropped from a tall bridge toward the water below. A second stone with mass = 0.75 kg is released from the same position 0.2 seconds later. Consider the system of two stones and treat the drop position as y = 0 m when answering the following.

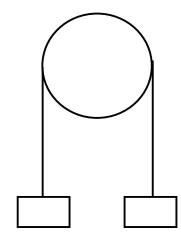
   a. What is the position of the center of mass when the second stone is initially dropped?
   b. What is the velocity of the center of mass when the second stone is initially dropped?
   c. What is the position of the center of mass when the first stone has fallen for 1.2 seconds?
   d. What is the velocity of the center of mass when the first stone has fallen for 1.2 seconds?
   e. What is the acceleration of the center of mass when the first stone has fallen for 1.2 seconds?

3. A 10 kg block is sliding along a frictionless surface at 4 m/s when it explodes into three pieces. A 2.5 kg piece leaves at 6 m/s with an angle of 30° to the original path. A 4 kg piece leaves at 4 m/s with an angle of 45° to the original path as shown. What is the final velocity of the remaining piece?



4. Two blocks (m_1= 2.0 kg and m_2= 4.0 kg) are connected over a pulley with a radius of 10 cm as shown and are released from rest at the same height.

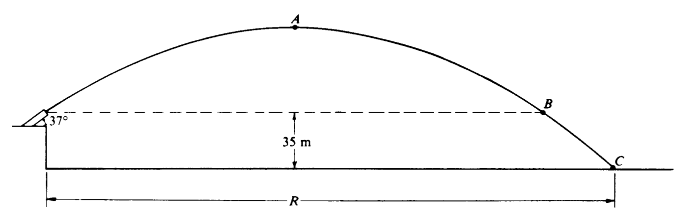
   a. What is the position of the center of mass for the blocks when they are initially released?
   b. What is the acceleration of the 4.0 kg block?
   c. What is the velocity of the center of mass 1.0 second after the blocks are released?
   d. What is the position of the center of mass 1.0 second after the blocks are released?



5. A 1000 kg cannon fires a 10 kg ball with a velocity of 100 m/s with respect to the cannon.

   a. What is the velocity of the ball with respect to the ground?
   b. What is the velocity of the cannon with respect to the ground?

6. A projectile is launched from the top of a cliff above level ground. At launch the projectile is 35 meters above the base of the cliff and has a velocity of 50 meters per second at an angle 37° with the horizontal. Air resistance is negligible. Consider the following two cases and use g = 10 m/s^2 sin 37° = 0.60, and cos 37° = 0.80.

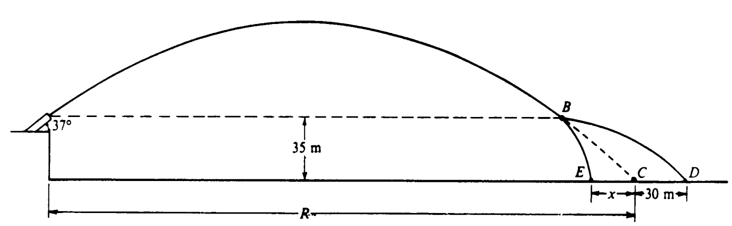


Case I: The projectile follows the path shown by the curved line in the following diagram.

a. Calculate the total time from launch until the projectile hits the ground at point C.
b. Calculate the horizontal distance R that the projectile travels before it hits the ground.
c. Calculate the speed of the projectile at points A, B and C.

Case II: A small internal charge explodes at point B in the following diagram, causing the projectile to separate into two parts of masses 6 kilograms and 10 kilograms. The explosive force on each part is horizontal and in the plane of the trajectory. The 6-kilogram mass strikes the ground at point D, located 30 meters beyond point C, where the projectile would have landed had it not exploded. The 10-kilogram mass strikes the ground at point E.

d. Calculate the distance x from C to *E.*

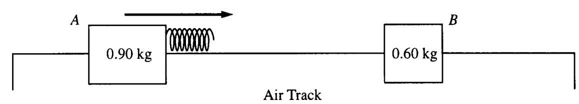


7. A 100 kg running back jumps to score a touchdown. He is traveling 8 m/s just before the goal line when he is hit by a 115 kg linebacker traveling 7 m/s in the opposite direction. They stick together following the collision.
a. What is the final speed of the combination?
b. What direction do they travel after the collision? (Does he score a touchdown?)
c. What type of collision is this?

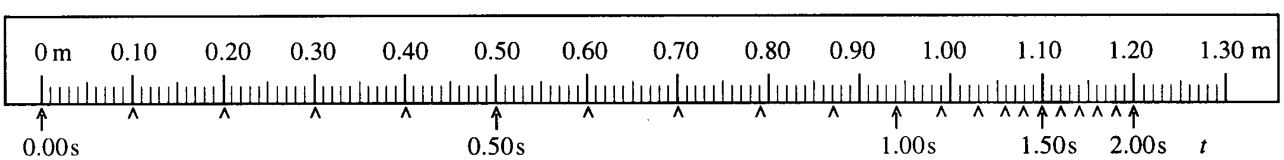
8. A 4 kg block is sliding along a frictionless surface at 5 m/s when it hits a 3 kg block that is traveling in the opposite direction at 2 m/s. After the collision, the 4 kg block is traveling 1 m/s in its original direction.
a. What is the initial momentum of the system?
b. What is the initial kinetic energy of the system?
c. What is the final velocity of the 3 kg block?
d. What is the final kinetic energy of the system?
e. What type of collision is this?

9. Two 1.5 kg blocks are traveling toward each other when they collide elastically. One is traveling 4 m/s to the right, while the other is traveling 3 m/s to the left.
a. What is the initial momentum of the system?
b. What is the initial kinetic energy of the system?
c. What is the final momentum of the system?
d. What is the final kinetic energy of the system?
e. What is the final velocity (speed and direction) of each block?

10. Two gliders move freely on an air track with negligible friction, as shown. Glider A has a mass of 0.90 kg and glider B has a mass of 0.60 kg. Initially, glider A moves toward glider B, which is at rest. A spring of negligible mass is attached to the right side of glider A. Strobe photography is used to record successive positions of glider A at 0.10 s intervals over a total time of 2.00 s, during which time it collides with glider B.

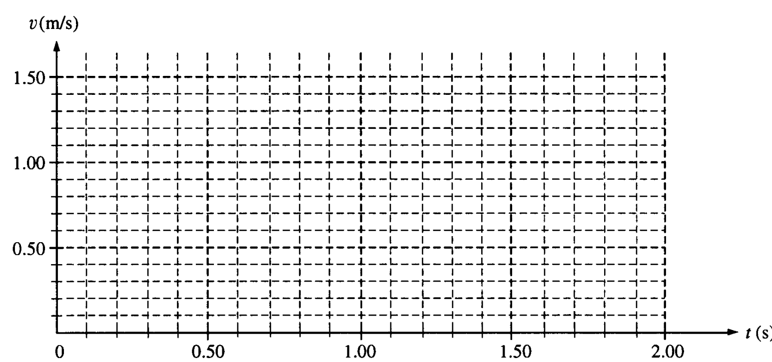


The following diagram represents the data for the motion of glider A. Positions of glider A at the end of each 0.10s interval are indicated by the symbol (^) against a metric ruler. The total elapsed time t after each 0.50 s is also indicated.



a. Determine the average speed of glider A for the following time intervals.
i. 0.10 s to 0.30 s ii. 0.90 s to 1.10 s iii. 1.70 s to 1.90 s

b. On the axes at right, sketch a graph, consistent with the data above, of the speed of glider A as a function of time t for the 2.00 s interval.



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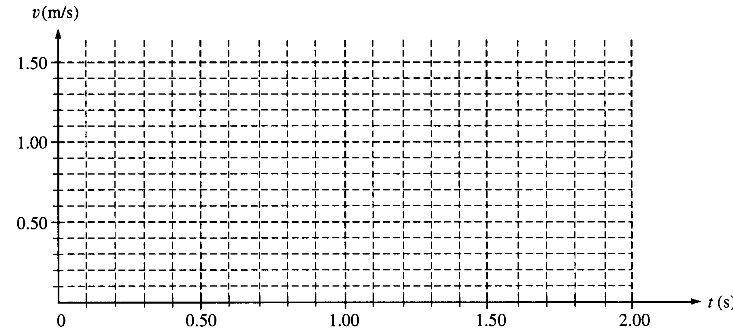
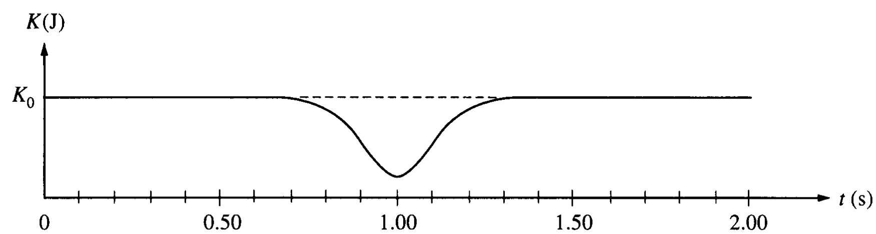
c. i. Use the data to calculate the speed of glider B immediately after it separates from the spring.
ii. On the axes at right, sketch a graph of the speed of glider B as a function of time t.

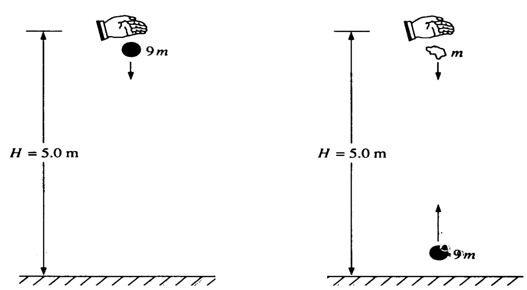


A graph of the total kinetic energy K for the two-glider system over the 2.00 s interval has the following shape. K_o is the total kinetic energy of the system at time t = 0.



d. i. Is the collision elastic? Justify your answer.
ii. Briefly explain why there is a minimum in the kinetic energy curve at t = 1.00 s.

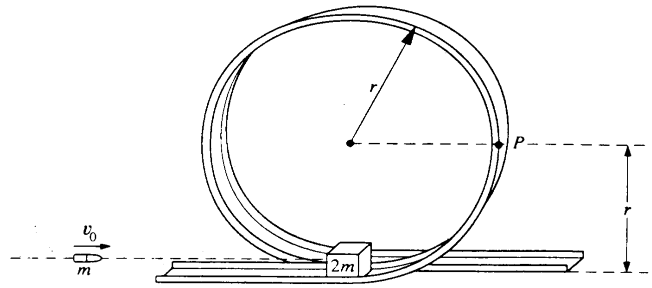
11. A ball of mass 9m is dropped from rest from a height H = 5.0 meters above the ground, as shown above on the left. It undergoes a perfectly elastic collision with the ground and rebounds. At the instant that the ball rebounds, a small blob of clay of mass m is released from rest from the original height H, directly above the ball, as shown above on the right. The clay blob, which is descending, eventually collides with the ball, which is ascending. Assume that $g = 10\ m/s^2$, that air resistance is negligible, and that the collision process takes negligible time.



a. Determine the speed of the ball immediately before it hits the ground.
b. Determine the time after the release of the clay blob at which the collision takes place.
c. Determine the height above the ground at which the collision takes place.
d. Determine the speeds of the ball and the clay blob immediately before the collision.
e. If the ball and the clay blob stick together on impact, what is the magnitude and direction of their velocity immediately after the collision?

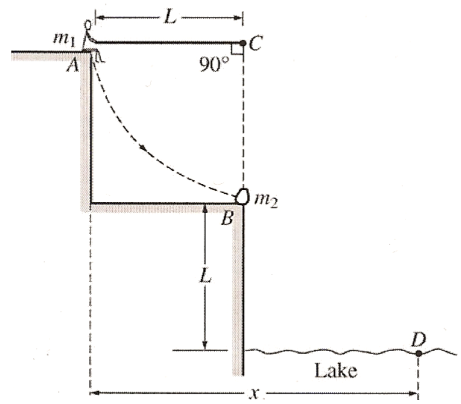
12. A crash test car of mass 1000 kg moving at a constant speed of 12 m/s collides completely inelastically with an object of mass M at time $t = 0$. The object was initially at rest. The speed v in m/s of the car-object system after the collision is given as a function of time t by the expression $v = \dfrac{8}{1+5t}$.
a. Calculate the mass M of the object.
b. Determine an expression for the resisting force on the car-object system after the collision as a function of time t.
c. Determine the impulse delivered to the car-object system from t=0 to t=2.0 s.

13. A small block of mass 2m initially rests on a track at the bottom of the circular, vertical loop-the-loop shown above, which has a radius r. The surface contact between the block and the loop is frictionless. A bullet of mass m strikes the block horizontally with initial speed v_o and remains embedded in the block as the block and bullet circle the loop. Determine each of the following in terms of m, v_0 r, and g.



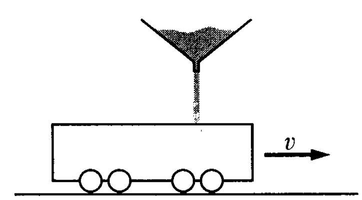
a. The speed of the block and bullet immediately after impact
b. The kinetic energy of the block and bullet when they reach point P on the loop
c. The minimum initial speed v_{min} of the bullet if the block and bullet are to successfully execute a complete circuit of the loop.

14. A rope of length L is attached to a support at point C. A person of mass m_1 sits on a ledge at position A holding the other end of the rope so that it is horizontal and taut, as shown. The person then drops off the ledge and swings down on the rope toward position B on the lower ledge where an object of mass m_2 is at rest. At position B the person grabs hold of the object and simultaneously lets go of the rope. The person and the object then land together in the lake at point D, which is a vertical distance L below position B. Air resistance and the mass of the rope are negligible. Derive expressions for each of the following in terms of m_1, m_2, L, and g.



a. The speed of the person just before the collision with the object.
b. The tension in the rope just before the collision with the object.
c. The speed of the person and the object just after the collision.
d. The ratio of the kinetic energy of the person-object system before the collision to the kinetic energy after the collision.
e. The total horizontal displacement x of the person from position A until the person and the object land in the water at point D.

15. An open-top railroad car (initially empty and of mass M_o) rolls with negligible friction along a straight horizontal track and passes under the spout of a sand conveyor. When the car is under the conveyor, sand is dispensed from the conveyor in a narrow stream at a steady rate $\Delta M/\Delta t = C$ and falls vertically from an average height h above the floor of the railroad car. The car has initial speed v_o and sand is filling it from time $t = 0$ to $t = T$. Express your answers to the following in terms of the given quantities and g.



a. Determine the mass M of the car plus the sand that it catches as a function of time t for $0 < t < T$.
b. Determine the speed v of the car as a function of time t for $0 < t < T$.
c. i. Determine the initial kinetic energy K_i of the empty car.
ii. Determine the final kinetic energy K_f of the car and its load.
iii. Is kinetic energy conserved? Explain why or why not.