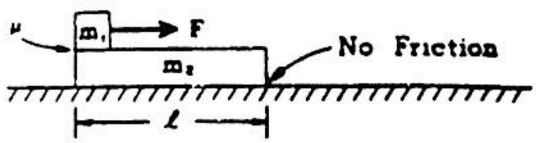
AP Physics Mechanics

Free Response Review Packet

Please do not write in this packet.

You may use $g = 10 m/s^2$ for these problems.

1. A horizontal force F is applied to a small block of mass m_1 to make It slide along the top of a larger block of mass m_2 and length l. The coefficient of friction between the blocks is μ. The larger block slides without friction along a horizontal surface. The blocks start from rest with the small block at one end of the larger block, as shown.

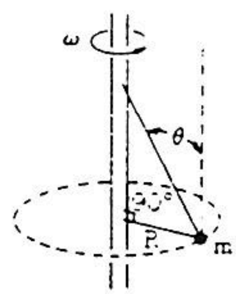


a. On the diagrams below draw all of the forces acting on each block. Identify each force.

b. Find the acceleration of each block. a_1 and a_2, relative to the horizontal surface.
c. In terms of l, a_1, and a_2, find the time t needed for the small block to slide off the end of the larger block.
d. Find an expression for the energy dissipated as heat because of the friction between the two blocks.

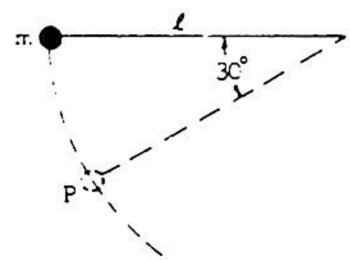
2. A 30-gram bullet is fired with a speed of 500 meters per second into a wall.
a. If the deceleration of the bullet is constant and it penetrates 12 centimeters into the wall, calculate the force on the bullet while it is stopping.
b. If the deceleration of the bullet is constant and it penetrates 12 centimeters into the wall, how much time is required for the bullet to stop?
c. Suppose, instead, that the stopping force increases from zero as the bullet penetrates. Discuss the motion in comparison to the case for a constant deceleration.

3. A ball of mass m is attached by two strings to a vertical rod. as shown. The entire system rotates at constant angular velocity ω about the axis of the rod.



a. Assuming ω is large enough to keep both strings taut, find the force each string exerts on the ball in terms of ω, m, g, R, and θ.
b. Find the minimum angular velocity, ω_{min} for which the lower string barely remains taut.

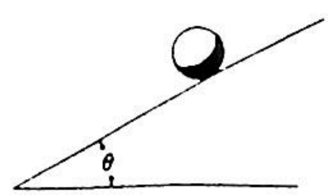
4. A pendulum consisting of a small heavy ball of mass m at the end of a string of length l is released from a horizontal position. When the ball is at point P, the string forms an angle of 30° with the horizontal as shown.



a. In the space below, draw a force diagram showing all of the forces acting on the ball at P. Identify each force clearly.

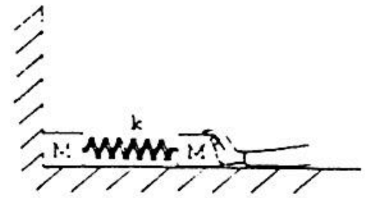
b. Determine the speed of the ball at P.
c. Determine the tension in the string when the ball is at P.
d. Determine the tangential acceleration of the ball at P.

5. The moment of inertia of a uniform uniform solid sphere (mass M, radius R) about a diameter is $2MR^2/5$. The sphere is placed on an inclined plane (angle θ) as shown and released from rest.



a. Determine the minimum coefficient of friction μ between the sphere and plane with which the sphere will roll down the incline without slipping.
b. If μ were zero, would the speed of the sphere at the bottom be greater, smaller, or the same as in part(a)? Explain your answer.

6. A system consists of two blocks, each of mass M, connected by a spring of force constant k. The system is initially shoved against a wall so that the spring is compressed a distance D from its original uncompressed length. The floor is frictionless. The system is now released with no initial velocity.



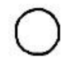
a. Determine the maximum speed of the right-hand block.

b. Determine the speed of the center of mass of the system when the left-hand block is no longer in contact with the wall.

c. Determine the period of oscillation for the system when the left-hand block is no longer in contact with the wall.

7. A sphere of mass m is released from rest. As it falls, the air exerts a retarding force on the sphere that is proportional to the sphere's velocity (FR = -kv). Neglect the buoyancy force of the air.

a. On the circles below draw vectors representing the forces acting on the sphere
   i. just after it is released and
ii. after it has been falling- for a long time and reached terminal velocity.
Give each vector a descriptive label

(i)  (ii) 

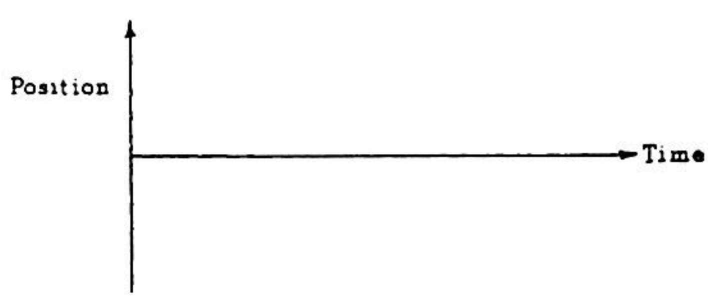
b. Determine the terminal velocity of the sphere.

c. Draw the following three graphs for the sphere's motion clearly showing significant features of the motion just after the sphere is released as well as after a long time.

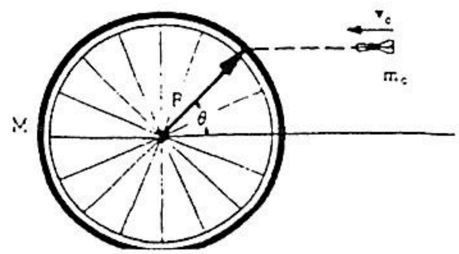
Acceleration as a function of time

ii. Velocity as a function of time

iii. Position as a function of time.



8. A bicycle wheel of mass M (assumed to be concentrated at its rim) and radius R is mounted horizontally so it may turn without friction on a vertical axle. A dart of mass m_o is thrown with velocity v_o as shown above and sticks in the tire.



a. If the wheel is initially at rest, find its angular velocity ω after the dart strikes.

b. In terms of the given quantities, determine the ratio:

$$\frac{\textit{final kinetic energy of the system}}{\textit{initial kinetic energy of the system}}$$

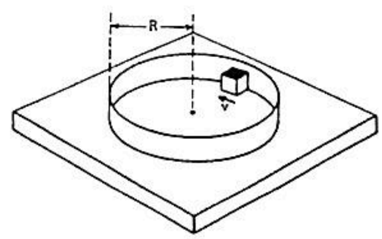
9. A uniform chain of mass M and length l hangs from a hook in the ceiling. The bottom link is now raised vertically and hung on the hook as shown above on the right.

a. Determine the increase in gravitational potential energy of the chain by considering the change in position of the center of mass of the chain.

b. Write an equation for the upward external force F(y) required to lift the chain slowly as a function of the vertical distance y.

c. Find the work done on the chain by direct integration of $\int Fdx$.

10. A small block of mass m slides on a horizontal frictionless surface as it travels around the inside of a hoop of radius R. The coefficient of friction between the block and the wall is μ; therefore, the speed v of the block decreases. In terms of m, R. μ, and v, find expressions for each of the following.

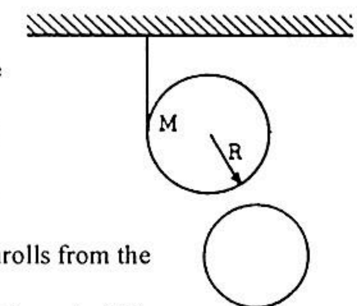


a. The frictional force on the block

b. The block's tangential acceleration dv/dt

c. The time required to reduce the speed of the block from an initial value v_0 to $v_o/3$

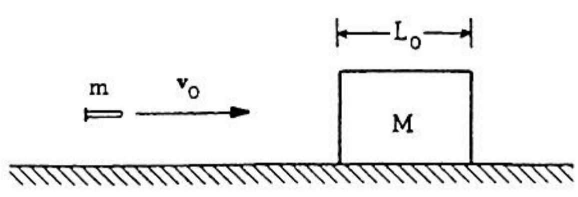
11. A cloth tape is wound around the outside of a uniform solid cylinder (mass M, radius R) and fastened to the ceiling as shown in the diagram above. The cylinder is held with the tape vertical and then released from rest. As the cylinder descends, it unwinds from the tape without slipping. The moment of inertia of a uniform solid cylinder about its center is $\frac{1}{2}MR^2$.



a. On the circle draw vectors showing all the forces acting on the cylinder after it is released. Label each force clearly.

b. In terms of g, find the downward acceleration of the center of the cylinder as it unrolls from the tape.

c. While descending, does the center of the cylinder move toward the left, toward the right, or straight down? Explain.



12. A bullet of mass m and velocity v_o is fired toward a block of thickness L_o and mass M. The block is initially at rest on a frictionless surface. The bullet emerges from the block with velocity $v_o/3$.

a. Determine the final speed of block M.

b. If, instead, the block is held fixed and not allowed to slide, the bullet emerges from the block with a speed $v_o/2$. Determine the loss of kinetic energy of the bullet

c. Assume that the retarding force that the block material exerts on the bullet is constant. In terms of L_o, what minimum thickness L should a fixed block of similar material have in order to stop the bullet?

d. When the block is held fixed, the bullet emerges from the block with a greater speed than when the block is free to move. Explain.

13. A block of mass m, which has an initial velocity v_o at time t = 0, slides on a horizontal surface.

a. How much work must be done on the block to bring it to rest?

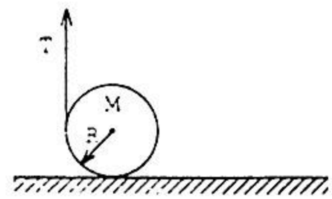
If the sliding friction force f exerted on the block by the surface is directly proportional to its velocity (that is, f = -Kv) determine the following:

b. The acceleration a of the block in terms of m, k, and v.

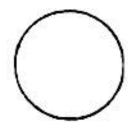
c. The speed v of the block as a function of time t.

d. The total distance the block slides.

14. A uniform cylinder of mass M, and radius R is initially at rest on a rough horizontal surface. The moment of inertia of a cylinder about its axis is $\frac{1}{2}MR^2$. A string, which is wrapped around the cylinder, is pulled upwards with a force T whose magnitude is 0.6Mg and whose direction is maintained vertically upward at all times. In consequence, the cylinder both accelerates horizontally and slips. The coefficient of kinetic friction is 0.5.



a. On the diagram below, draw vectors that represent each of the forces acting on the cylinder identify and clearly label each force.

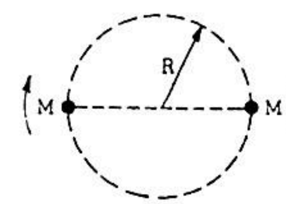


b. Determine the linear acceleration a of the center of the cylinder.

c. Calculate the angular acceleration α of the cylinder.

d. Your results should show that a and αR are not equal. Explain.

15. Two stars each of mass M form a binary star system such that both stars move in the same circular orbit of radius R. The universal gravitational constant is G.

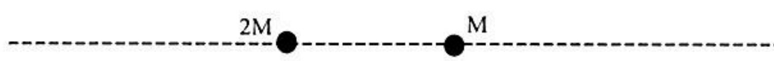


a. Use Newton's laws of motion and gravitation to find an expression for the speed v of either star in terms of R, G, and M.

b. Express the total energy E of the binary star system in terms of R, G, and M.

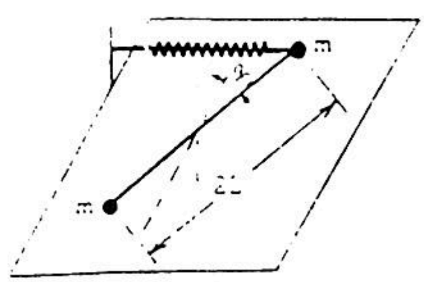
Suppose instead, one of the stars had a mass 2M.

c. On the following diagram, show circular orbits for this star system.



d. Find the ratio of the speeds, v_{2M}/v_{M}.

16. A stick of length 2L and negligible mass has a point mass m affixed to each end. The stick is arranged so that it pivots in a horizontal plane about a frictionless vertical axis through its center. A spring of force constant k is connected to one of the masses as shown above. The system is in equilibrium when the spring and stick are perpendicular. The stick is displaced through a small angle θ_o as shown and then released from rest at t = 0 .



a. Determine the restoring torque when the stick is displaced from equilibrium through the small angle θ_o.

b. Determine the magnitude of the angular acceleration of the stick just after it has been released.

c. Write the differential equation whose solution gives the behavior of the system after it has been released.

d. Write the expression for the angular displacement θ of the stick as a function of time t after it has been released from rest.

17. An amusement park ride consists of a ring of radius A from which hang ropes of length l with seats for the riders as shown in Figure I. When the ring is rotating at a constant angular velocity ω each rope forms a constant angle θ with the vertical as shown in Figure II. Let the mass of each rider be m and neglect friction, air resistance, and the mass of the ring, ropes, and seats.

a. In the space below, draw and label all the forces acting on rider X (represented by the point below) under the constant rotating condition of Figure II. Clearly define any symbols you introduce.

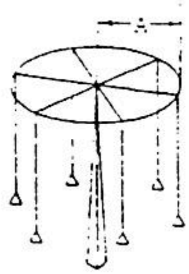


Figure I

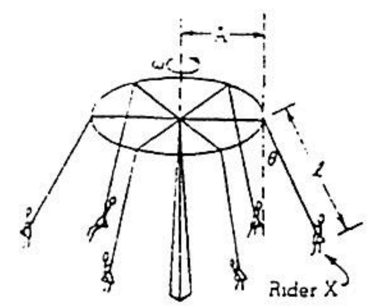


Figure II

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b. Derive an expression for ω in terms of A, l, θ and the acceleration of gravity g.

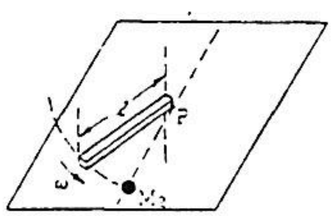
c. Determine the minimum work that the motor that powers the ride would have to perform to bring the system from rest to the constant rotating condition of Figure II. Express your answer in terms of m, g, l, θ, and the speed v of each rider.

18. A system consists of a mass M_2 and a uniform rod of mass M_1 and length l. The rod is initially rotating with an angular speed ω on a horizontal frictionless table about a vertical axis fixed at one end through point P. The moment of inertia of the rod about P is $Ml^2/3$. The rod strikes the stationary mass M_2. As a result of this collision, the rod is stopped and the mass M_2 moves away with speed v.

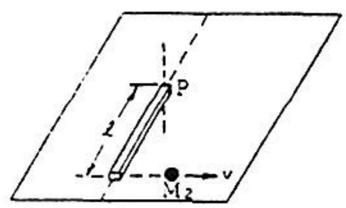
a. Using angular momentum conservation determine the speed v in terms of M_1, M_2, l, and ω.

b. Determine the linear momentum of this system just before the collision in terms of M_1, l, and ω.

c. Determine the linear momentum of this system just after the collision in terms of M_1 l, and ω.



Before Collision



After Collision

d. What is responsible for the change in the linear momentum of this system during the collision?

e. Why is the angular momentum of this system about point P conserved during the collision?

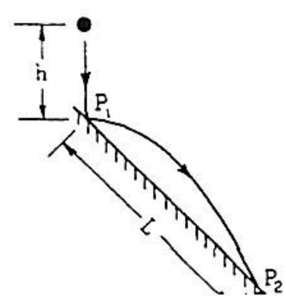
19. A ball of mass m is released from rest at a distance h above a frictionless plane inclined at an angle of 45° to the horizontal as shown above. The ball bounces elastically off the plane at point P_1 and strikes the plane again at point P_2. In terms of g and h determine each of the following quantities:

a. The velocity (a vector) of the ball just after it first bounces off the plane at P_1.

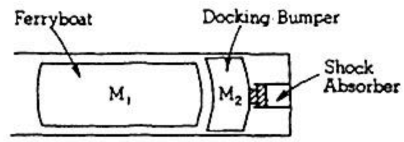
b. The time the ball is in flight between points P_1 and P_2.

c. The distance L along the plane from P_1 to P_2.

d. The speed of the ball just before it strikes the plane at P_2.

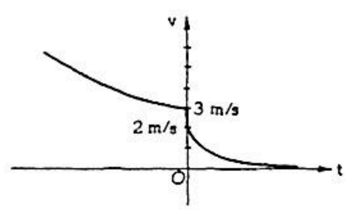


20. A ferryboat of mass $M_1 = 2.0 \times 10^5$ kilograms moves toward a docking bumper of mass M_2 that is attached to a shock absorber. Shown below is a speed v vs. time t graph of the ferryboat from the time it cuts off its engines to the time it first comes to rest after colliding with the bumper. At the instant it hits the bumper, t = 0 and v = 3 meters per second.



a. After colliding inelastically with the bumper, the ferryboat and bumper move together with an initial speed of 2 meters per second. Calculate the mass of the bumper M_2.

b. After colliding, the ferryboat and bumper move with a speed given by the expression $v = 2e^{-4t}$. Although the boat never comes precisely to rest, it travels only a finite distance. Calculate that distance.



c. While the ferryboat was being slowed by water resistance before hitting the bumper, its speed was given by $1/v = 1/3 + \beta t$, where β is a constant. Find an expression for the retarding force of the water on the boat as a function of speed.

21. A mass m constrained to move on a frictionless horizontal surface is attached to a frictionless peg by a massless spring having force constant k. The unstretched length of the spring is l_1, as shown in Figure 1. When the mass moves in a circle about the peg with constant angular velocity ω_o, the length of the spring is l_2 as shown in Figure 2. Express your answers to parts a, b, and c in terms of m, k, ω_o and l_1.

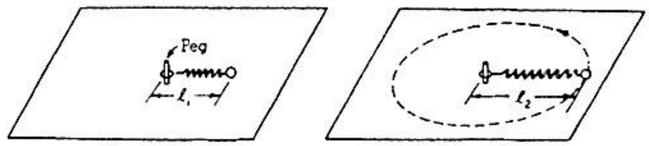


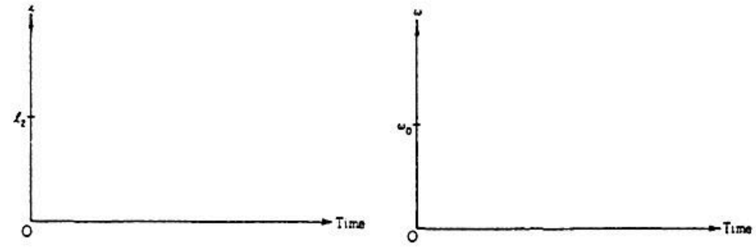
Figure 1 Figure 2

a. Determine the length l_2

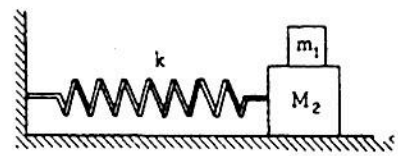
b. Assume the total energy of the system in Figure 1 is zero. Determine the total energy of the rotating system in Figure 2.

c. Determine the magnitude of the angular momentum of the system.

d. While the mass is rotating about the peg with angular velocity ω_o, it is struck by a hammer that provides a small impulse directed inward. On the axes below, sketch graphs to indicate qualitatively the manner in which the length of the spring l and the angular velocity ω will vary with time in the subsequent motion.



22. A small mass m_1 rests on but is not attached to a large mass M_2 that slides on its base without friction. The maximum frictional force between m_1 and M_2 is f. A spring of spring constant k is attached to the large mass M_2 and to the wall as shown above.



a. Determine the maximum horizontal acceleration that M_2 may have without causing m_1 to slip.

b. Determine the maximum amplitude A for simple harmonic motion of the two *masses if* they are to move together, i.e., m_1 must not slip on M_2.

c. The two-mass combination is pulled to the right the maximum amplitude A found in part (b) and released. Describe the frictional force on the small mass m_1 during the first half cycle of oscillation.

d. The two-mass combination is now pulled to the right a distance of A' greater than A and released.

i. Determine the acceleration of m_1 at the instant the masses are released.

ii. Determine the acceleration of M_2 at the instant the masses are released.

23. A block of mass m slides at velocity v_o across a horizontal frictionless surface toward a large curved movable ramp of mass 3m as shown in Figure 1. The ramp, initially at rest, also can move without friction and has a smooth circular frictionless face up which the block can easily slide. When the block slides up the ramp, it momentarily reaches a maximum height as shown in Figure II and then slides back down the frictionless face to the horizontal surface as shown in Figure III.

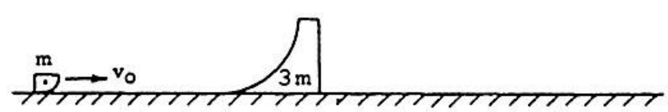


Figure I

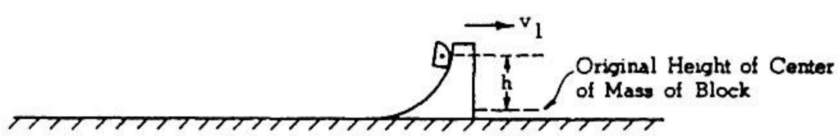


Figure II

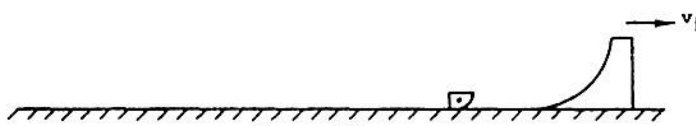


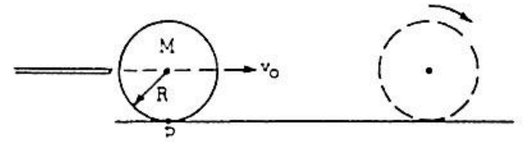
Figure III

a. Find the velocity v_1 of the moving ramp at the instant the block reaches its maximum height.

b. To what maximum height h does the center of mass of the block rise above its original height?

c. Determine the final speed v_f of the ramp and the final speed v' of the block after the block returns to the level surface. State whether the block is moving to the right or to the left.

24. A billiard ball has mass M, radius R, and moment of inertia about the center of mass $I_c = 2\ MR^2/5$. The ball is struck by a cue stick along a horizontal line through the ball's center of mass so that the ball initially slides with a velocity v_o as shown above. As the ball moves across the rough billiard table (coefficient of sliding friction μ_k), its motion gradually changes from pure translation through rolling with slipping to rolling without slipping.



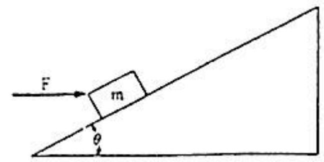
a. Develop an expression for the linear velocity v of the center of the ball as a function of time while it is rolling with slipping.

b. Develop an expression for the angular velocity ω of the ball as a function of time while it is rolling with slipping.

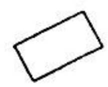
c. Determine the time at which the ball begins to roll without slipping.

d. When the ball is struck it acquires an angular momentum about the fixed point P on the surface of the table. During the subsequent motion the angular momentum about point P remains constant despite the frictional force. Explain why this is so.

25. A block of mass m, acted on by a force of magnitude F directed horizontally to the right as shown above, slides up an inclined plane that makes an angle θ with the horizontal. The coefficient of sliding friction between the block and the plane is μ.



a. On the diagram of the block, draw and label all the forces that act on the block as it slides up the plane.



b. Develop an expression in terms of m, θ, F, μ, and g, for the block's acceleration up the plane.

c. Develop an expression for the magnitude of the force F that will allow the block to slide up the plane with constant velocity. What relation must θ and μ satisfy in order for this solution to be physically meaningful?

26. A swing seat of mass M is connected to a fixed point P by a massless cord of length L. A child also of mass M sits on the seat and begins to swing with zero velocity at a position at which the cord makes a 60° angle with the vertical is shown in Figure I. The swing continues down until the cord is exactly vertical at which time the child jumps off in a horizontal direction. The swing continues in the same direction until its cord makes a 45° angle with the vertical as shown in Figure II: at that point it begins to swing in the reverse direction. With what velocity relative to the ground did the child leave the swing? ($\cos 45° = \sin 45° = \sqrt{2}/2$, $\sin 30° = \cos 60° = 1/2$, $\cos 30° = \sin 60° = \sqrt{3}/2$

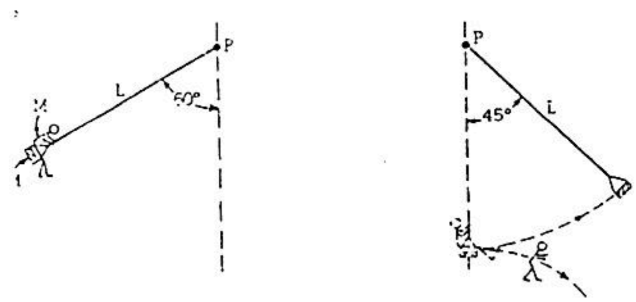


Figure I

Figure II

27. A thin, uniform rod of mass M_1 and length L , is initially at rest on a frictionless horizontal surface. The moment of inertia of the rod about its center of mass is $M_1L^2/12$. As shown in Figure I, the rod is struck at point P by a mass m_2 whose initial velocity v is perpendicular to the rod. After the collision, mass m_2 has velocity $-\frac{1}{2}v$ as shown in Figure II. Answer the following in terms of the symbols given.

a. Using the principle of conservation of linear momentum, determine the velocity v' of the center of mass of this rod after the collision.

b. Using the principle of conservation of angular momentum, determine the angular velocity ω of the rod about its center of mass after the collision.

c. Determine the change in kinetic energy of the system resulting from the collision.

Views From Above

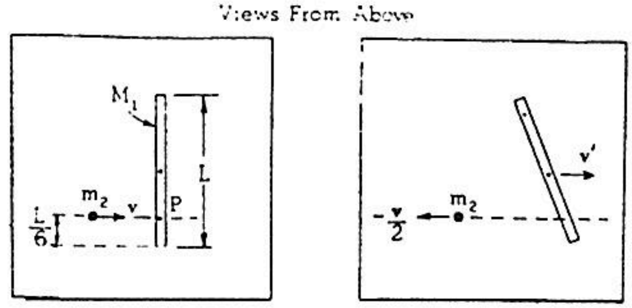
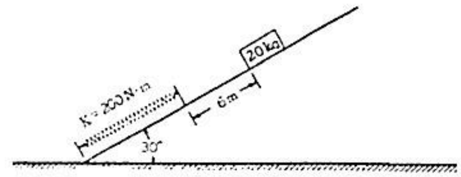


Figure I: Before

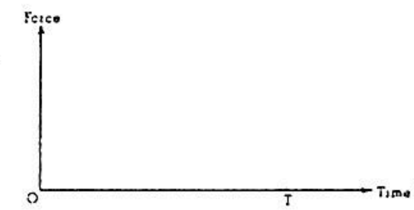
Figure II: After

28. A 20 kg mass, released from rest, slides 6 meters down a frictionless plane inclined at an angle of 30° with the horizontal and strikes a spring of spring constant K = 200 newtons/meter as shown in the diagram above. Assume that the spring is ideal, that the mass of the spring is negligible, and that mechanical energy is conserved. Use $g = 10\ m/s^2$, ($\sin 30° = ½$, $\cos 30° = 0.866$)



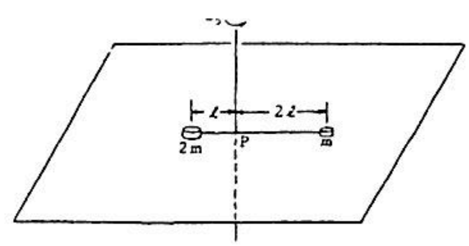
a. Determine the speed of the block just before it hits the spring.
b. Determine the distance the spring has been compressed when the block comes to rest.
c. Is the speed of the block a maximum at the instant the block strikes the spring? Justify your answer.

29. A car of mass M moves with an initial speed v_o on a straight horizontal road. The car is brought to rest by braking in such a way that the speed of the car is given as a function of time t by $v = (v_o^2 - Rt/M)^{½}$



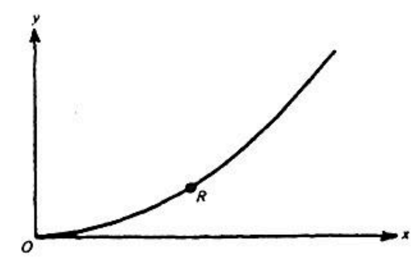
a. Develop an equation that expresses the time rate of change of kinetic energy.
b. Determine the time it takes to bring the car to a complete stop.
c. Develop an equation for the acceleration of the car as a function of time t.
d. On the axes, sketch the magnitude of the braking force as a function of time t

30. A system consists of two small disks, of masses m and 2m, attached to a rod of negligible mass of length $3l$ as shown above. The rod is free to turn about a vertical axis through point P. The two disks rest on a rough horizontal surface; the coefficient of friction between the disks and the surface is μ. At time t = 0, the rod has an initial counterclockwise angular velocity ω_o about P. The system is gradually brought to rest by friction. Develop expressions for the following quantities in terms of μ m, l, g, and ω_o



a. The initial angular momentum of the system about the axis through P
b. The frictional torque acting on the system about the axis through P
c. The time T at which the system will come to rest

31. A particle moves along the parabola with equation $y = (1/2)x^2$ shown.

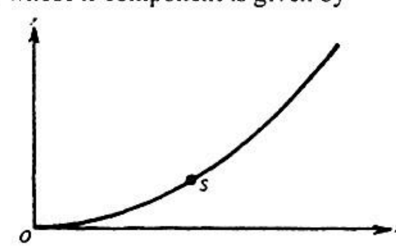


a. Suppose the particle moves so that the x-component of its velocity has the constant value $v_x = C$; that is, $x = Ct$

i. On the diagram above, indicate the directions of the particle's velocity vector **v** and acceleration vector **a** at point R, and label each vector.
ii. Determine the y-component of the particle's velocity as a function of x.
iii. Determine the y-component of the particle's acceleration.

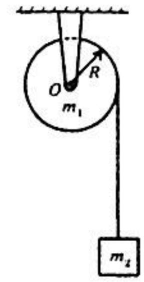
b. Suppose, instead, that the particle moves along the same parabola with a velocity whose x-component is given by

$$v_x = \frac{C}{\sqrt{1 + x^2}}$$



i. Show that the particle's speed is constant in this case.
ii. On the diagram, indicate the directions of the particle's velocity vector **v** and acceleration vector **a** at point S, and label each vector. State the reasons for your choices.

32. A uniform solid cylinder of mass m_1 and radius R is mounted on frictionless bearings about a fixed axis through O. The moment of inertia of the cylinder about the axis is $I = \frac{1}{2}m_1R^2$. A block of mass m_2, suspended by a cord wrapped around the cylinder as shown above, is released at time t = 0.



a. On the diagrams draw and identify all of the forces acting on the cylinder and on the block.

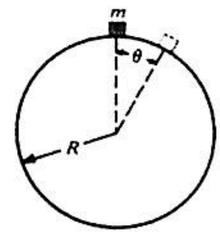
b. In terms of m_1, m_2, R. and g, determine each of the following.

i. The acceleration of the block

ii. The tension in the cord

iii. The angular momentum of the disk as a function of time t.

33. A particle of mass m slides down a fixed, frictionless sphere of radius R. starting from rest at the top.



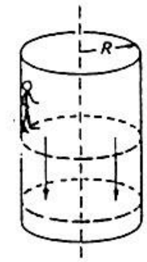
a. In terms of m, g, R. and θ, determine each of the following for the particle while it is sliding on the sphere.

i. The kinetic energy of the particle

ii. The centripetal acceleration of the mass

iii. The tangential acceleration of the mass

b. Determine the value of θ at which the particle leaves the sphere.



34. An amusement park ride consists of a rotating vertical cylinder with rough canvas walls. The floor is initially about halfway up the cylinder wall as shown above. After the rider has entered and the cylinder is rotating sufficiently fast, the floor is dropped down, yet the rider does not slide down. The rider has mass of 50 kilograms, the radius R of the cylinder is 5 meters, the angular velocity of the cylinder when rotating is 2 radians per second, and the coefficient of static friction between the rider and the wall of the cylinder is 0.6.

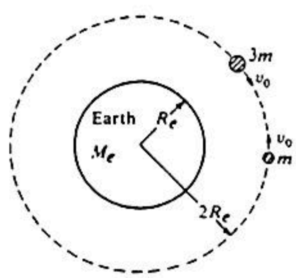
a. On the diagram. draw and identify the forces on the rider when the system is rotating and the floor has dropped down.

b. Calculate the centripetal force on the rider when the cylinder is rotating and state what provides that force.

c. Calculate the upward force that keeps the rider from falling when the floor is dropped down and state what provides that force.

d. At the same rotational speed, would a rider of twice the mass slide down the wall? Explain your answer.

35. Two satellites, of masses m and 3m, respectively, are in the same circular orbit about the Earth's center, as shown in the diagram above. The Earth has mass M_e and radius R_e. In this orbit, which has a radius of $2R_e$, the satellites initially move with the same orbital speed v_o but in opposite directions.



a. Calculate the orbital speed v_o of the satellites in terms of G, M_e, and R_e.

b. Assume that the satellites collide head-on and stick together. In terms of v_o find the speed v of the combination immediately after the collision.

c. Calculate the total mechanical energy of the system immediately after the collision in terms of G, m, M_e, and R_e. Assume that the gravitational potential energy of an object is defined to be zero at an infinite distance from the Earth.

36. A small body of mass m located near the Earth's surface falls from rest in the Earth's gravitational field. Acting on the body is a resistive force of magnitude kmv, where k is a constant and v is the speed of the body.

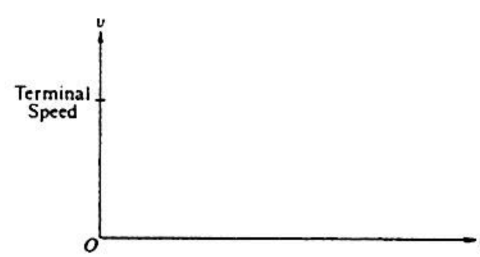
a. On the diagram below, draw and identify all of the forces acting on the body as it falls.

b. Write the differential equation that represents Newton's second law for this situation.

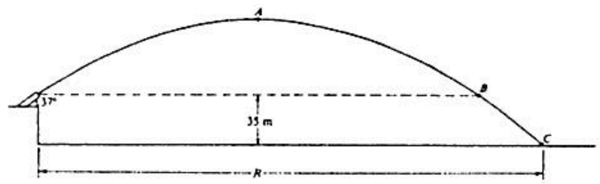
c. Determine the terminal speed v_T of the body.

d. Integrate the differential equation once to obtain an expression for the speed v as a function of time t. Use the condition that v = 0 when t= 0.

e. On the axes provided below, draw a graph of the speed v as a function of time t.



37. A projectile is launched from the top of a cliff above level ground. At launch the projectile is 35 meters above the base of the cliff and has a velocity of 50 meters per second at an angle 37° with the horizontal. Air resistance is negligible. Consider the following two cases and use g = 10 m/s₂ sin 37° = 0.60, and cos 37° = 0.80.



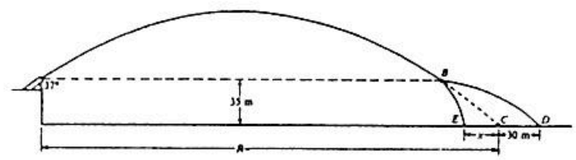
Case I: The projectile follows the path shown by the curved line in the following diagram.

a. Calculate the total time from launch until the projectile hits the ground at point C.

b. Calculate the horizontal distance R that the projectile travels before it hits the ground.

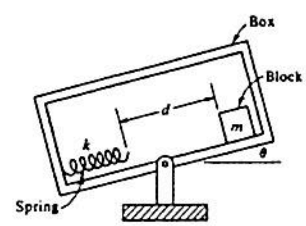
c. Calculate the speed of the projectile at points A, B and C.

Case II: A small internal charge explodes at point B in the following diagram, causing the projectile to separate into two parts of masses 6 kilograms and 10 kilograms. The explosive force on each part is horizontal and in the plane of the trajectory. The 6-kilogram mass strikes the ground at point D, located 30 meters beyond point C, where the projectile would have landed had it not exploded. The 10-kilogram mass strikes the ground at point E.



d. Calculate the distance x from C to *E*.

38. An apparatus to determine coefficients of friction is shown above. The box is slowly rotated counterclockwise. When the box makes an angle θ with the horizontal, the block of mass m just starts to slide, and at this instant the box is stopped from rotating. Thus at angle θ, the block slides a distance d, hits the spring of force constant k, and compresses the spring a distance x before coming to rest. In terms of the given quantities, derive an expression for each of the following.

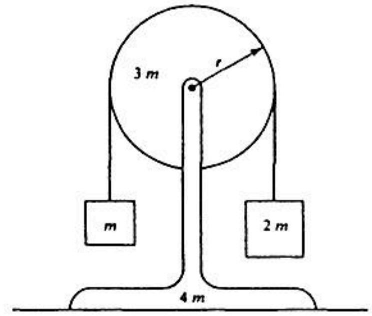


a. μ_s the coefficient of static friction.

b. ΔE, the loss in total mechanical energy of the block-spring system from the start of the block down the incline to the moment at which it comes to rest on the compressed spring.

c. μ_k, the coefficient of kinetic friction.

39. A pulley of mass 3m and radius r is mounted on frictionless bearings and supported by a stand of mass 4m at rest on a table as shown above. The moment of inertia of this pulley about its axis is $1.5mr^2$.



Passing over the pulley is a massless cord supporting a block of mass m on the left and a block of mass 2m on the right. The cord does not slip on the pulley, so after the block-pulley system is released from rest, the pulley begins to rotate.

a. On the diagrams, draw and label all the forces acting on each block.

b. Use the symbols identified in part (a) to write each of the following.

i. The equations of translational motion (Newton's second law) for each of the two blocks

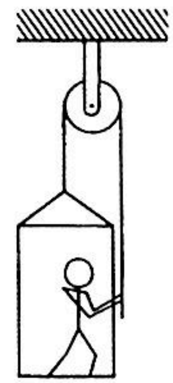
ii. The analogous equation for the rotational motion of the pulley

c. Solve the equations in part (b) for the acceleration of the two blocks.

d. Determine the tension in the segment of the cord attached to the block of mass m.

e. Determine the normal force exerted on the apparatus by the table while the blocks are in motion.

40. The figure shows an 80-kilogram person standing on a 20-kilogram platform suspended by a rope passing over a stationary pulley that is free to rotate. The other end of the rope is held by the person. The masses of the rope and pulley are negligible. You may use $g = 10\ m/s^2$. Assume that friction is negligible, and the parts of the rope shown remain vertical.



a. If the platform and the person are at rest, what is the tension in the rope?

The person now pulls on the rope so that the acceleration of the person and the platform is $2\ m/s^2$ upward.

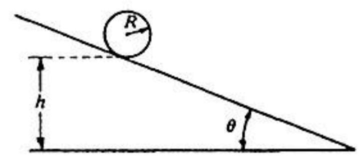
b. What is the tension in the rope under these new conditions?

c. Under these conditions, what is the force exerted by the platform on the person?

After a short time, the person and the platform reach and sustain an upward velocity of 0.4 m/s.

d. Determine the power output of the person required to sustain this velocity.

41. An inclined plane makes an angle of θ with the horizontal, as shown above. A solid sphere of radius R and mass M is initially at rest in the position shown, such that the lowest point of the sphere is a vertical height h above the base of the plane. The sphere is released and rolls down the plane without slipping. The moment of inertia of the sphere about an axis through its center is $2MR^2/5$. Express your answers in terms of M, R. h, g, and θ.



a. Determine the following for the sphere when it is at the bottom of the plane:
    i. Its translational kinetic energy
    ii. Its rotational kinetic energy

b. Determine the following for the sphere when it is on the plane.
    i. Its linear acceleration
    ii. The magnitude of the frictional force acting on it

The solid sphere is replaced by a hollow sphere of identical radius R and mass M. The hollow sphere, which is released from the same location as the solid sphere, rolls down the incline without slipping.

c. What is the total kinetic energy of the hollow sphere at the bottom of the plane?

d. State whether the rotational kinetic energy of the hollow sphere is greater than, less than, or equal to that of the solid sphere at the bottom of the plane. Justify your answer.

42. A special spring is constructed in which the restoring force is in the opposite direction to the displacement, but is proportional to the *cube of the displacement;* i.e., $F = -kx^3$

This spring is placed on a horizontal frictionless surface. One end of the spring is fixed, and the other end is fastened to a mass M. The mass is moved so that the spring is stretched a distance A and then released. Determine each of the following in terms of k, A, and M.

a. The potential energy in the spring at the instant the mass is released

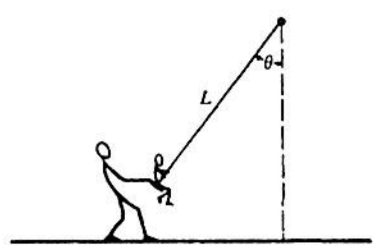
b. The maximum speed of the mass

c. The displacement of the mass at the point where the potential energy of the spring and the kinetic energy of the mass are equal

The amplitude of the oscillation is now increased:

d. State whether the period of the oscillation increases, decreases, or remains the same. Justify your answer.

43. An adult exerts a horizontal force on a swing that is suspended by a rope of length L, holding it at an angle θ with the vertical. The child in the swing has a weight W and dimensions that are negligible compared to L. The weights of the rope and of the seat are negligible. In terms of W and θ, determine



a. the tension in the rope;

b. the horizontal force exerted by the adult.

The adult releases the swing from rest. In terms of W and θ determine

c. the tension in the rope just after the release (the swing is instantaneously at rest);

d. the tension in the rope as the swing passes through its lowest point.

44. The above graph shows the potential energy U(x) of a particle as a function of its position x.

a. Identify all points of equilibrium for this particle.

Suppose the particle has a constant total energy of 4.0 joules, as shown by the dashed line on the graph.

b. Determine the kinetic energy of the particle at the following positions

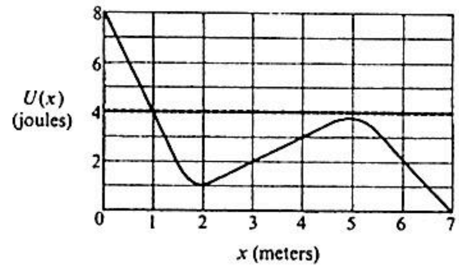
i. x = 2.0 m

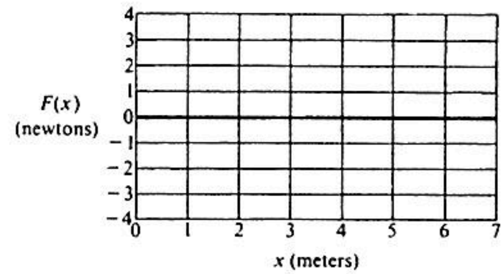
ii. x = 4.0 m

c. Can the particle reach the position x = 0.5 m? Explain.

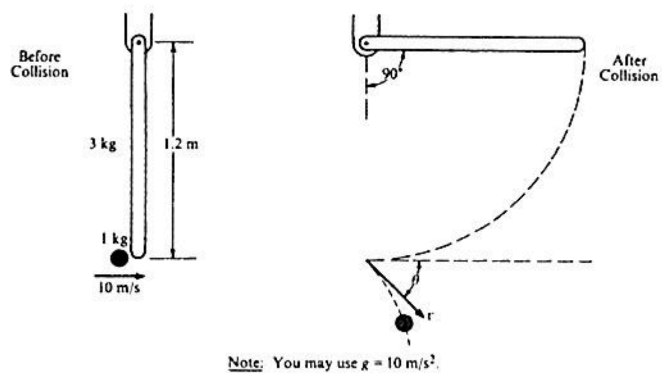
d. Can the particle reach the position x = 5.0 m? Explain.

e. On the grid below, carefully draw a graph of the conservative force acting on the particle as a function of x, for $0 < x < 7$ meters.





45. A 1.0-kilogram object is moving horizontally with a velocity of 10 meters per second, as shown above, when it makes a glancing collision with the lower end of a bar that was hanging vertically at rest before the collision. For the system consisting of the object and bar, linear momentum is not conserved in this collision, but kinetic energy is conserved. The bar, which has a length l of 1.2 meters and a mass m of 3.0 kilograms, is pivoted about the upper end. Immediately after the collision the object moves with speed v at an angle θ relative to its original direction. The bar swings freely, and after the collision reaches a maximum angle of 90° with respect to the vertical. The moment of inertia of the bar about the pivot is $I_{bar} = ml^2/3$ Ignore all friction.



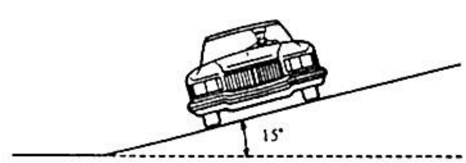
a. Determine the angular velocity of the bar immediately after the collision.

b. Determine the speed v of the 1-kilogram object immediately after the collision.

c. Determine the magnitude of the angular momentum of the object about the pivot just before the collision.

d. Determine the angle θ.

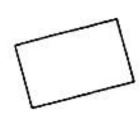
46. A highway curve that has a radius of curvature of 100 meters is banked at an angle of 15° as shown above.



a. Determine the vehicle speed for which this curve is appropriate if there is no friction between the road and the tires of the vehicle.

On a dry day when friction is present, an automobile successfully negotiates the curve at a speed of 25 m/s.

b. On the diagram below, in which the block represents the automobile, draw and label all of the forces on the automobile.



c. Determine the minimum value of the coefficient of friction necessary to keep this automobile from sliding as it goes around the curve.