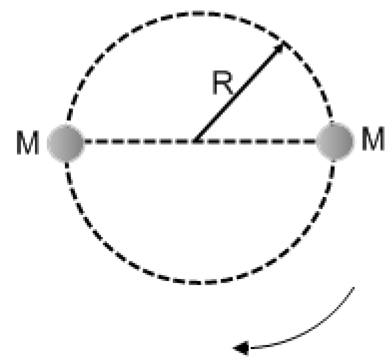
AP Physics C

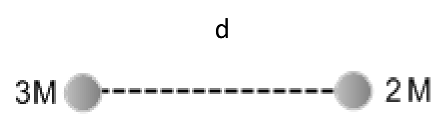
Gravity

Free Response Problems

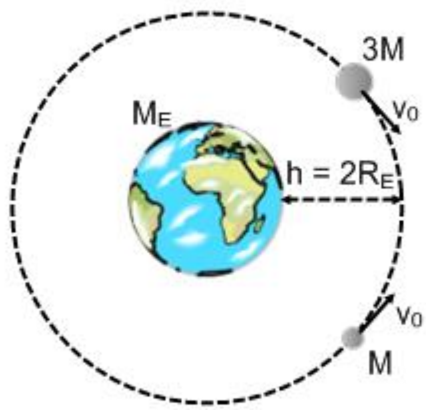


1. Two identical stars from a binary star system move in the same circular orbit of radius R. Each star has a mass M. The universal gravitational constant is G.
    a. Find the orbital speed of either star in terms of M, R, and G.
    b. Find the expression for the total energy of the binary star system in terms of R, M, and G.

   Two new stars with masses 3M and 2M form another star system with a distance d between their centers.

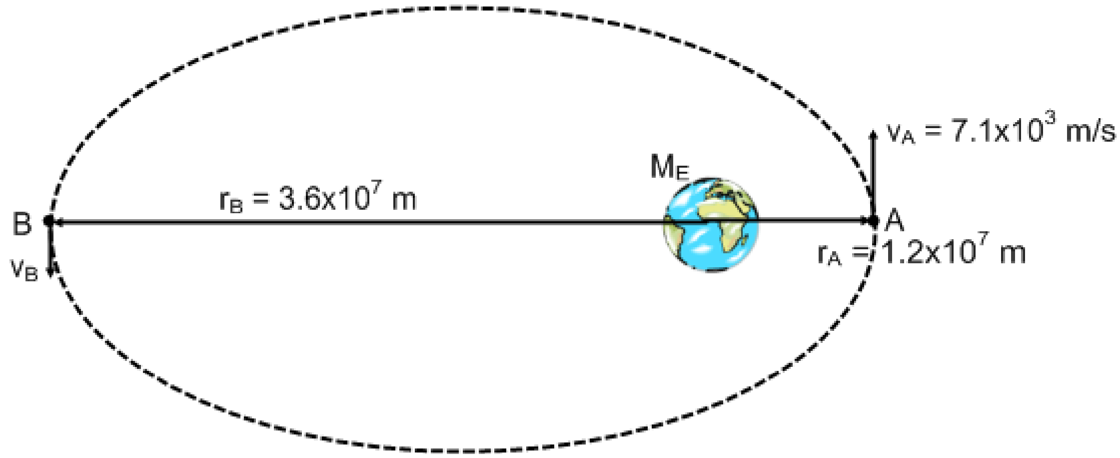


    c. On the diagram above show the circular orbits for the star system.
    d. Find the ratio of the speeds V_{3M}/V_{2M}.



2. Two satellites of masses M and 3M move in the same circular orbit around Earth at a distance h = $2R_E$ above the Earth's surface. The Earth's mass is M_E and the Earth's radius is R_E. The satellites move with the same initial speed V_0 in opposite directions.
   a. Find the orbital speed V_0 of the satellites in terms of M_E, R_E, and G.
   b. Find the orbital period T of the satellites in terms of M_E, R_E, and G.

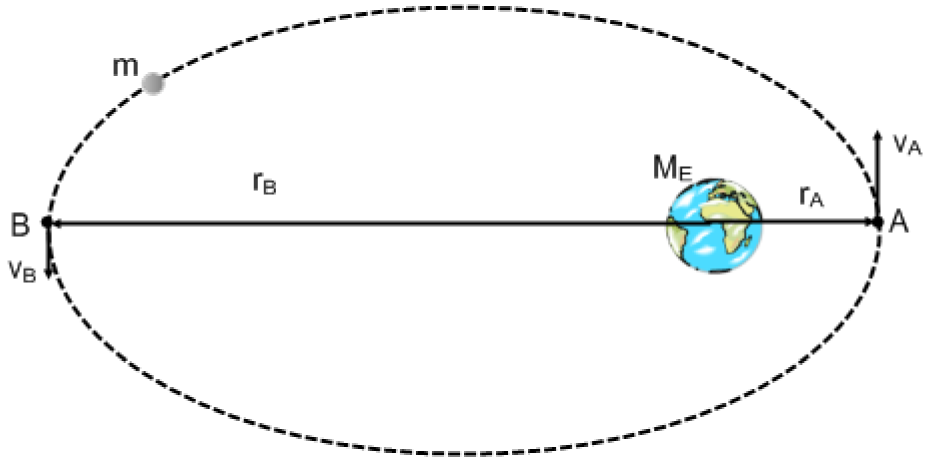
   The satellites collide and stick together.
   c. Find the speed of the satellites just after the collision.
   d. Assuming that the gravitational potential energy of an object is zero at an infinite distance from the Earth; find the total energy of the system just after the collision.



3. A satellite of mass 2,000 kg is in an elliptical orbit about the Earth. When the satellite reaches point A, which is the closest point to the Earth, its orbital radius is 1.2×10^7 m and its orbital velocity is 7.1×10^3 m/s. ($M_E = 6 \times 10^{24}$ kg and $R_E = 6.4 \times 10^6$ m)
   a. Determine the total mechanical energy of the satellite at point A, assuming that the gravitational potential energy is zero at an infinite distance from the Earth.
   b. Determine the angular momentum of the satellite at point A.
   c. What is the minimum speed of the satellite at point A in order to escape from Earth?

When the satellite reaches point B, which is the furthest point from the Earth, its orbital radius is 3.6×10^7 m.

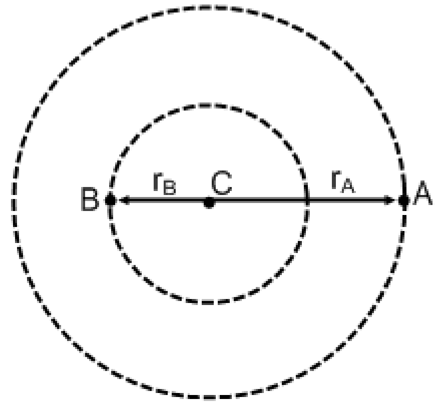
   d. Determine the speed of the satellite at point B.



4. A satellite of mass m is in an elliptical orbit around the Earth, which has mass M_E and radius R_E. The orbital radius varies from the smallest value r_A at point A to the largest value r_B at point B. The satellite has a velocity v_A at point A. Assume that the gravitational potential energy $U_g = 0$ when the satellite is at an infinite distance from the Earth. Present all the answers in terms of m, M_E, R_E, r_A, r_B, and v_A.
    a. Derive an expression for the gravitational potential energy of the satellite as a function of distance r from the center of the Earth by using an appropriate definite integral.
    b. Determine the total mechanical energy of the satellite when it is at point A.
    c. Determine the angular momentum of the satellite with respect to the center of the Earth when it is at point A.
    d. Determine the velocity of the satellite when it is at point B.

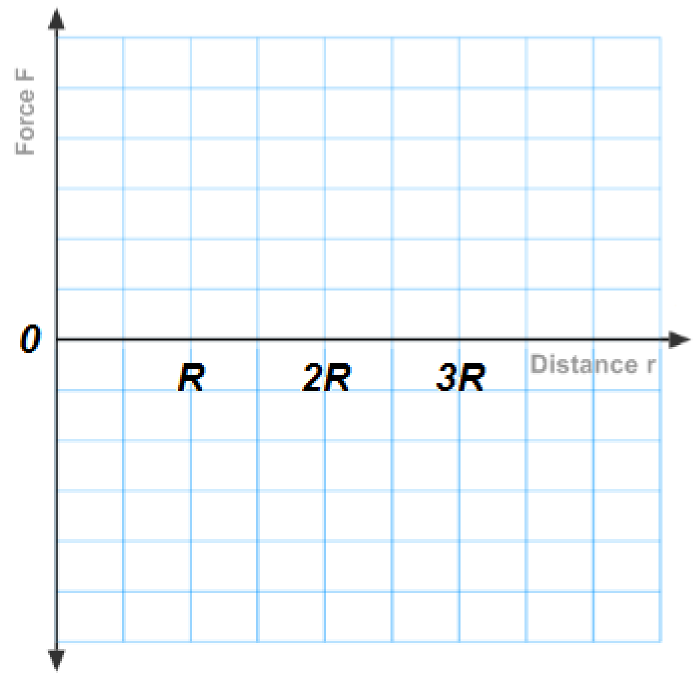
When the satellite passes point A it makes a new maneuver that results in the orbit changing from elliptical to circular with a radius r_A.

e. Determine the new orbital velocity of the satellite.
f. Determine the work done by the satellite's engine to change the orbit.



5. Two stars A and B move in circular orbits around their common center of mass at point C. Each star is revolving with the same period T. Show all results in terms of T, r_A, r_B, and fundamental constants.
   a. Determine the centripetal acceleration of star A.
   b. Determine the mass of star A.
   c. Determine the mass of star B.
   d. Determine the moment of inertia of the system of two stars with respect to the center of mass.
   e. Determine the angular momentum of the system of two stars with respect to the center of mass.

6. Suppose we drill a hole through the Earth along its diameter and drop a small mass m down the hole. Assume that the Earth is not rotating and has a uniform density throughout its volume. The Earth's mass is M_E and its radius is R_E. Let r be the distance from the falling object to the center of the Earth.
    a. Derive an expression for the gravitational force on the small mass as a function of r when it is moving inside the Earth.
    b. Derive an expression for the gravitational force on the small mass as a function of r when it is outside the Earth.
    c. On the diagram below, draw the gravitational force on the small mass as a function of its distance r from the center of the Earth.



    d. Determine the work done by the gravity as the mass moves from the surface to the center.
    e. Determine the speed of the mass at the center of the Earth if the Earth has a given density ρ.
    f. Determine the time it takes the mass to move from the surface to the center.

7. A satellite is placed into a circular orbit around Mars, which has a mass $M = 6.4\times10^{23}$ kg and radius $R = 3.4\times10^{6}$ m.
    a. Derive an expression for the orbital speed assuming that the orbital radius is R.
    b. Derive an expression for the orbital period assuming that the orbital radius is R.
    c. The orbital period of the satellite can be synchronized with Mars' rotation, whose period is 24.6 hours. Determine the required orbital radius for this to occur.
    d. Determine the escape speed from the surface of Mars.

8. It is known that Jupiter has 63 moons and that some of the moons are very small with a radius of about 1-2 km. The table below represents the circular orbital data of four of the largest moons of Jupiter.

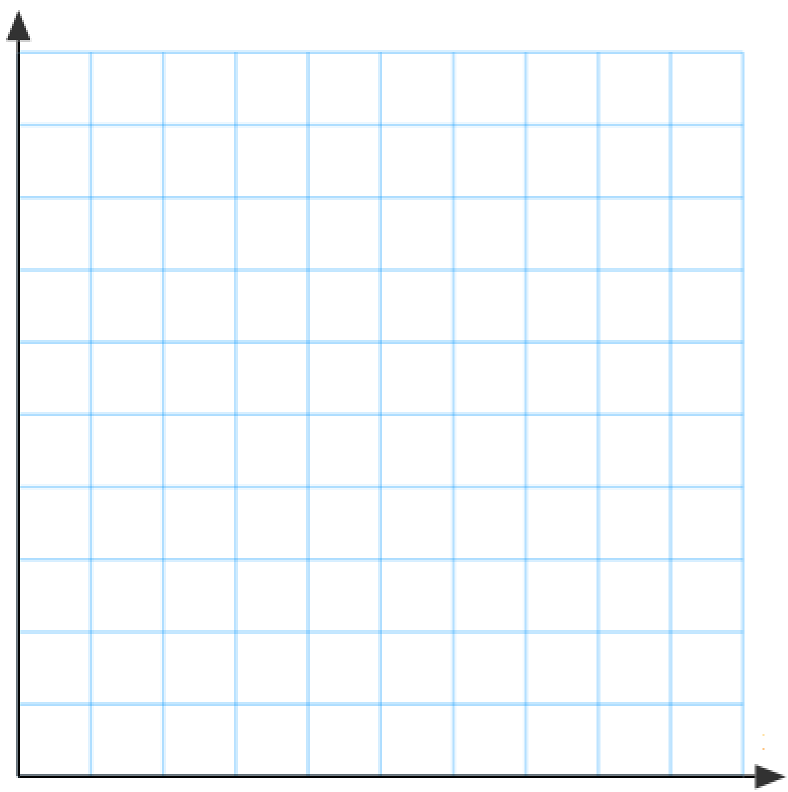
Moon	Orbital Radius(m)	Orbital Period (s)		
Io	4.22×10^{8}	1.53×10^{5}		
Europa	6.71×10^{8}	3.07×10^{5}		
Ganymedy	1.07×10^{9}	6.18×10^{5}		
Callisto	1.88×10^{9}	1.44×10^{6}		

a. Derive an expression for the orbital period of one of Jupiter's moon as a function of the orbital radius r if it has a mass M.

b. Which quantities should be graphed in order to have a straight line whose slope could be used to find Jupiter's mass?

c. Complete the table by calculating those two quantities. Label the top of each column and show the units.

d. Plot the graph on the diagram below. Label the axes and show corresponding numbers.

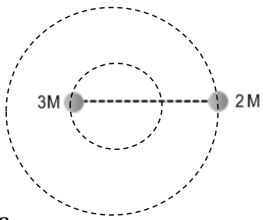


e. Find the mass of Jupiter from the graph.

Free Response Answers

1. a. $\sqrt{\frac{GM}{4R}}$

   b. $-\frac{GM^2}{4R}$

   c. 

   d. $\frac{2}{3}$

2. a. $\sqrt{\frac{GM_E}{3R_E}}$

   b. $2\pi\sqrt{\frac{27{R_E}^3}{GM}}$

   c. $\frac{\sqrt{GM_E/3R_E}}{2}$

   d. $-\frac{7GM_E m}{6R_E}$

3. a. $-1.63 * 10^{10}\ J$

   b. $1.704 * 10^{14}\ kg \cdot m^2/s$

   c. $8167\frac{m}{s}$

   d. $2366.67\frac{m}{s}$

4. a. $-\frac{GM_E m}{r}$

b. $\frac{mv_A{}^2}{2} - \frac{GM_E m}{r_A}$

c. $mv_A r_A$

d. $\frac{v_A r_A}{v_B}$

e. $\sqrt{\frac{GM_E}{r_A}}$

f. $\frac{m}{2}(\frac{GM_E}{r_A} - v_A^2)$

5. a. $\frac{4\pi^2 r_A}{T^2}$

b. $\frac{4\pi^2 r_B (r_A + r_B)^2}{GT^2}$

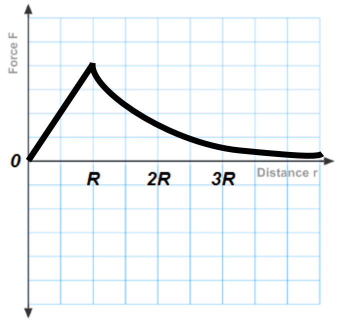
c. $\frac{4\pi^2 r_A (r_A + r_B)^2}{GT^2}$

d. $\frac{4\pi^2 r_B r_A (r_A + r_B)^3}{GT^2}$

6. a. $\frac{GM_E m}{R_E^3} * r$

b. $\frac{GM_E m}{r^2}$

c.



d. $\frac{GM_E m}{2R_E}$

e. $2R_E\sqrt{\frac{G\rho\pi}{3}}$

f. $2\pi\sqrt{\frac{R_E^2}{GM_E}}$

7. a. $\sqrt{\frac{GM}{R}}$

b. $2\pi\sqrt{\frac{R^3}{GM}}$

c. $2.04 * 10^7\ m$

d. $5011.05\frac{m}{s}$

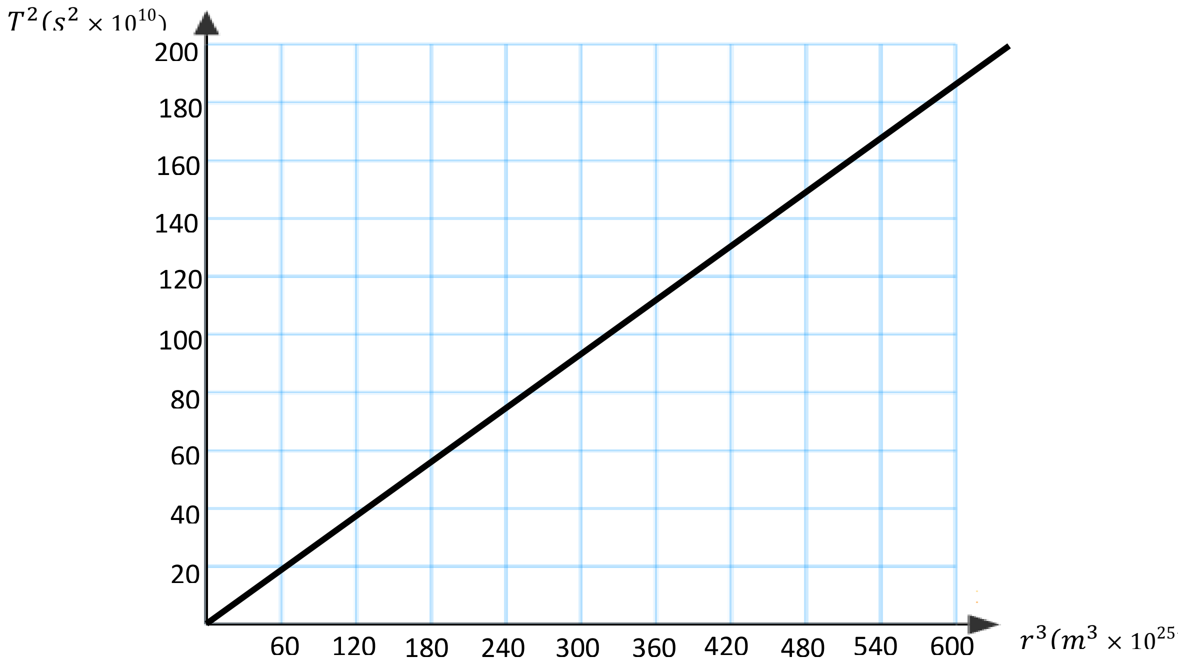
8. a. $2\pi\sqrt{\frac{r^3}{GM}}$

b. $T^2 as\ a\ function\ of\ r^3$

c.

Moon	Orbital Radius(m)	Orbital Period (s)	T^2 (s^2)	r^3 (m^3)
Io	4.22×10^8	1.53×10^5	2.34×10^{10}	7.52×10^{25}
Europa	6.71×10^8	3.07×10^5	9.42×10^{10}	3.02×10^{26}
Ganymedy	1.07×10^9	6.18×10^5	3.82×10^{11}	1.23×10^{27}
Callisto	1.88×10^9	1.44×10^6	2.07×10^{12}	6.64×10^{27}

d.



e. $1.78 * 10^{27}\ kg$