Physics C Exam - Mechanics 1998 Solution

	BASIC IDEA	SOLUTION	ANSWER
#1.	$W = \overrightarrow{F} \cdot \overrightarrow{d}$	$W = Fd \cos \theta$	В
#2	projectile motion	v_h remains constant because there is no force in that direction v_v is zero, that is why it goes no higher $a = g$ because gravity is still acting on it as a net force.	Е
#3	$v = \frac{dx}{dt}$	Reading the given graph says the slope of the postion graph must start at zero and increase to a maximum for the first section. The slope then remains constant and positive for the second section, and finally, the slope remains positive but decrease to zero at the end of the graph.	D
#4	$\vec{F} = \vec{ma}$ $a = \frac{dv}{dt}$ and $v = \frac{dx}{dt}$	$x = t^3 - 6t^2 + 9t$ $v = \frac{dx}{dt} = 3t^2 - 12t + 9$ $a = \frac{dv}{dt} = 6t - 12$ the F = m(6t-12) = 0 when t = 2s	В
#5	$\tau = r_{\perp}F$	Torque is a vector. Lets call clockwise negative and counter clockwise positiv -3R(2F) + 2R(F) + 3R(F) + 3R(F) = 2RF	ve C
#6.	$v_t = r \omega$ no slipping $v_t = v_{cm}$ p = mv	$p = Mv = MR\omega$ (note these equations deal only with the magnitudes of the quantities)	А
#7.	$\vec{F} = -G\frac{Mm}{r^2}\hat{r}$ Universal Lay of Gravitation	Force of gravitation is pulls toward the asteroid haver as the distances increases the force decreases with the inverse square.	А
#8.	$\vec{F} = -G \frac{Mm}{r^2} \hat{r}$	At the highest point $r = 2R$ so the force is one-fouth of the value at	D
#9.	$\vec{F} = \vec{ma}$ $F = -kx$ $ma = -kx$ $a = -\frac{k}{m}x$ $\omega = \sqrt{\frac{k}{m}}$ $T = \frac{2\pi}{m}$	the surface of the asteroind and the acceleration must also be one-fourth Here $\sqrt{\frac{k}{m}} = \sqrt{9} = 3$ \therefore T = $\frac{2\pi}{3}$	D
#10.	$T = 2\pi \sqrt{\frac{I}{g}}$	squaring the equation gives $T^2 = 4\pi^2 \frac{1}{g}$ so if T is doubled T^2 is quadrupled and since everything else on the right is constant g must be one fourth as great.	А
#11.	$v = r\omega$ $a_{T} = \frac{dv_{T}}{dt}$ $a = \frac{v^{2}}{r}$	solved this equation gives $\omega v/R$ ^v _T is the speed and is constant therefore $a_T = 0$ here v is the speed (v _T), again constant and r = R a consant	E
#12.	$\mathbf{F} = \frac{\Delta \mathbf{p}}{\Delta t}$	$F\Delta t = \Delta p$, so Δp equals the area bounded by the force function on a F vs. t graph. Here the positive areas and the negative areas are of equal magnitude and add to zero.	C 1

#13 Cons. of Momentum
$$p_m + p_{2m} = p_{3m}$$
 C
 $mv + 2m(v2) = 3mv_f$
 $2mv = 3mv$ therefore $v_f = \frac{2}{3}mv_f$
 $2mv = 3mv$ therefore $v_f = \frac{2}{3}mv_f$
 $2mv = 3mv$ therefore $v_f = \frac{2}{3}mv_f$
#14. $F = -kx$ $F = -100((.03) = -3$ N. Magnitude is 3N B
#15. $W = \int_{0}^{R} r ds$ Since F is perpendicular to the instantaneous displacement ds and any moment A
#16. conservation of energy $\Delta U + \Delta K = 0$
 $(3U_0 - U_0) + (0 - \frac{1}{2}mv^2)$ $\therefore v = \sqrt{\frac{4U_0}{m}}$ C
 $(3U_0 - U_0) + (0 - \frac{1}{2}mv^2)$ $\therefore v = \sqrt{\frac{4U_0}{m}}$
#17. $F = -\frac{dU}{dr}$ $F = -(-\frac{3}{2}vr\frac{5}{2}) = \frac{3}{2}vr\frac{5}{2}$ A
#18. conservation of energy $U_1 + K_1 = U_1 + K_f$
 $101 + 0 = 51 + K$ $\therefore K = 51$
#19. $W_R = \Delta K$ Resulting force on the elevatior is the force of the cable minus the force of
gravity (weight) on the elevator: $11,000 - 1,0000(98) = 1.200N$ therefore the
work of the routlant force is $1,200N$ (Sm cos 180° = -9.600 J and this must
equal the change in kinetic energy. $9,600 = K_f - K_i = 0 - \frac{1}{2}mv^2 = -500 v^2$.
It then follows that $v^2 = 19.2$ and $v = \sqrt{19.2}$ which is about 4.4 m/s.
#20. $F = G\frac{Mm}{r^2}$ $F = G\frac{MM}{D^2} = M\frac{v^2}{D} = 2M\frac{v^2}{D}$ B
 $F' = ma$ $GM_{f2D} = v^2$
 $a = \frac{v^2}{r}$
#21. $F' = \mu N$ $\mu = \frac{F}{N} = \frac{1}{mg} - \frac{f}{rsin\phi}$ E
 $F' = ma$ $r_{new}^2 must = 2r^2$ and it follows that $r_{new} = \sqrt{2}r$
components max $r_{new}^2 must = 2r^2$ and it follows that $r_{new} = \sqrt{2}r$
#23. This question was not counted when the exam was scored -
#24. Newtork's 3 Law No net external forces on the center of mass does not accelerate. $v_i = 0$ therefore v remains zero.
#25. $v = \frac{d}{t}$ Since the speed is constant the dancer will take equal time intervals for each acc.

#26.
$$s = \frac{1}{2}at^2 + v_it$$

Applying this equation to the horizontal motion: $x = v_h t$

С

separating variables gives
$$\frac{dv}{g-bv} = dt$$
 then

$$\int_{0}^{v} \frac{dv}{g-bv} = \int_{0}^{t} dt \quad \text{substituting } u = g \text{-bv gives}$$

$$\int_{0}^{g-bv} \frac{-1}{b} \int_{g}^{du} \frac{du}{u} = t \quad \text{integrating gives } \frac{-1}{b} [\ln(g-bv) - \ln g] = t$$

$$\ln(1 - \frac{b}{g}v) = -bt \quad \text{and } (1 - \frac{b}{g}v) = e^{-bt} \quad \text{and } v = \frac{g}{b} (1 - e^{-bt})$$

#35. Conservation of Energy $U_0 + K_0 = U_A + K_A$ Where "o" indicates the equilibrium position $U = \frac{1}{2} kx^2$ and "A" indicates maximum amplitude. Substituting into this equation $K = \frac{1}{2} mv^2$ we have: $0 + \frac{1}{2} Mv_m^2 = \frac{1}{2} kA^2 + 0$ \therefore $k = \frac{Mv_m^2}{A^2}$

D